$\boxed{1}$ (12 points) Compute the derivatives of the following functions. Do not simplify your answers.

(a) (4 points) $g(x) = (3x+1)\sec\sqrt{2-7x}$

$$g'(x) = 3\cdot\sec\sqrt{2-7x} + (3x+1)\cdot\sec\sqrt{2-7x}\cdot\tan\sqrt{2-7x}\cdot\frac{-7}{2\sqrt{2-7x}}$$

(b) (4 points) $h(x) = \sin\left(2+\cos\sqrt{1+x^3}\right)$

$$h'(x) = \cos\left(2+\cos\sqrt{1+x^3}\right)\cdot\left(-\sin\sqrt{1+x^3}\right)\cdot\frac{3x^2}{2\sqrt{1+x^3}}$$

(c) (4 points) $y = \left(\tan^{-1}x\right)^{\ln x}$

$$\begin{aligned}
y &= \left(\tan^{-1}x\right)^{\ln x}\\
\ln y &= \ln\left(\tan^{-1}x\right)^{\ln x}\\
\ln y &= \ln x\cdot\ln\left(\tan^{-1}x\right)\\
\frac{1}{y}\cdot\frac{dy}{dx} &= \frac{1}{x}\cdot\ln\left(\tan^{-1}x\right)+\ln x\cdot\frac{1}{\tan^{-1}x}\cdot\frac{1}{1+x^2}\\
\frac{dy}{dx} &= \left(\tan^{-1}x\right)^{\ln x}\cdot\left[\frac{1}{x}\cdot\ln\left(\tan^{-1}x\right)+\ln x\cdot\frac{1}{\tan^{-1}x}\cdot\frac{1}{1+x^2}\right]
\end{aligned}$$

$\boxed{2}$ (12 points) Consider the curve given by the parametric equations

$$x = t^3 - 7t + 5,\ y = 4t^3 - 3t^2 + 10t$$

(a) (6 points) Find the equation of the tangent line to the curve when $t = -1$.

$x(-1) = 11 \qquad y(-1) = -17$

$\dfrac{dx}{dt} = 3t^2 - 7 \qquad \left.\dfrac{dx}{dt}\right|_{t=-1} = -4$

$\dfrac{dy}{dt} = 12t^2 - 6t + 10 \qquad \left.\dfrac{dy}{dt}\right|_{t=-1} = 28$

$\left.\dfrac{dy}{dx}\right|_{t=-1} = -\dfrac{28}{4} = -7$

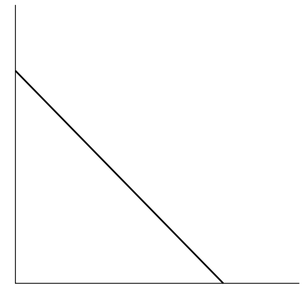
$y + 17 = -7(x - 11)$

(b) (6 points) Find all times t when the tangent line has slope 5.

From part (a) we have $\dfrac{dy}{dx} = \dfrac{12t^2 - 6t + 10}{3t^2 - 7}$.

$$\begin{aligned}
\frac{12t^2 - 6t + 10}{3t^2 - 7} &= 5\\
12t^2 - 6t + 10 &= 15t^2 - 35\\
0 &= 3t^2 + 6t - 45\\
0 &= t^2 + 2t - 15\\
0 &= (t-3)(t+5)\\
t &= -5, 3
\end{aligned}$$

$\boxed{3}$ (10 points) A ladder 17 ft long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 0.5 ft/s. Consider the triangle formed by the ladder, the ground and the wall. At what rate is the area of this triangle changing when the bottom of the ladder is 8 ft from the wall?



Let y be the height of the top of the ladder.

Let x be the distance of the bottom of the ladder from the wall.

Let A be the area of the triangle.

We know that $\dfrac{dx}{dt} = 0.5$.

We wish to compute $\dfrac{dA}{dt}$ *when* $x = 8$.

We have $A = \dfrac{1}{2}xy$.

By the Pythagorean Theorem, we have $y = \sqrt{289 - x^2}$

Thus $A = \dfrac{1}{2}x\sqrt{289 - x^2}$.

Differentiating with respect to t gives

$$\begin{aligned}
A &= \frac{1}{2}x\sqrt{289 - x^2} \\
\frac{dA}{dt} &= \frac{1}{2} \cdot \frac{dx}{dt} \cdot \sqrt{289 - x^2} + \frac{1}{2} \cdot x \cdot \frac{-2x}{2\sqrt{289 - x^2}} \cdot \frac{dx}{dt} \\
&= \frac{1}{2} \cdot 0.5 \cdot \sqrt{289 - 8^2} + \frac{1}{2} \cdot 8 \cdot \frac{-2 \cdot 8}{2\sqrt{289 - 8^2}} \cdot 0.5 \\
&= -\frac{161}{60} \\
&\approx -2.68 \ ft^2/sec
\end{aligned}$$

$\boxed{4}$ (8 points) Use a linear approximation to estimate $\sqrt[5]{31}$.

Let $f(x) = x^{1/5}$ *and take* $a = 32$.

$$\begin{aligned} f(32) &= 2 \\ f'(x) &= \frac{1}{5}x^{-4/5} \\ f'(32) &= \frac{1}{80} \\ L(x) &= 2 + \frac{1}{80}(x-32) \\ \sqrt[5]{31} &\approx L(31) \\ &= 2 + \frac{1}{80}(31-32) \\ &= \frac{159}{80} \end{aligned}$$

$\boxed{5}$ (8 points) Find all the points (a, b) on the curve $2x^3 + y^2 + y = 0$ where the tangent line is vertical.

$$\begin{aligned} 2x^3 + y^2 + y &= 0 \\ 6x^2 + 2yy' + y' &= 0 \\ y' &= -\frac{6x^2}{2y+1} \end{aligned}$$

Thus the tangent line is vertical when $y = -\dfrac{1}{2}$.

$$\begin{aligned} 2x^3 + y^2 - 8y &= 0 \\ 2x^3 + (-1/2)^2 + (-1/2) &= 0 \\ x^3 &= \frac{1}{8} \\ x &= \frac{1}{2} \end{aligned}$$

The only point on the curve with a vertical tangent is $(1/2, -1/2)$.