$\boxed{1}$ (12 points) Compute the derivatives of the following functions. Do not simplify your answers.

(a) (4 points) $g(x) = (2x+1)\sec\sqrt{3-5x}$

$$g'(x) = 2\cdot\sec\sqrt{3-5x} + (2x+1)\cdot\sec\sqrt{3-5x}\cdot\tan\sqrt{3-5x}\cdot\frac{-5}{2\sqrt{3-5x}}$$

(b) (4 points) $h(x) = \sin\left(2+\sin\sqrt{1+x^3}\right)$

$$h'(x) = \cos\left(2+\sin\sqrt{1+x^3}\right)\cdot\cos\sqrt{1+x^3}\cdot\frac{3x^2}{2\sqrt{1+x^3}}$$

(c) (4 points) $y = \left(\tan^{-1}x\right)^{\ln x}$

$$\begin{aligned}
y &= \left(\tan^{-1}x\right)^{\ln x}\\
\ln y &= \ln\left(\tan^{-1}x\right)^{\ln x}\\
\ln y &= \ln x\cdot\ln\left(\tan^{-1}x\right)\\
\frac{1}{y}\cdot\frac{dy}{dx} &= \frac{1}{x}\cdot\ln\left(\tan^{-1}x\right) + \ln x\cdot\frac{1}{\tan^{-1}x}\cdot\frac{1}{1+x^2}\\
\frac{dy}{dx} &= \left(\tan^{-1}x\right)^{\ln x}\cdot\left[\frac{1}{x}\cdot\ln\left(\tan^{-1}x\right) + \ln x\cdot\frac{1}{\tan^{-1}x}\cdot\frac{1}{1+x^2}\right]
\end{aligned}$$

$\boxed{2}$ (12 points) Consider the curve given by the parametric equations

$$x = t^3 - 7t + 5, \ y = 2t^3 - 3t^2 + 3t$$

(a) (6 points) Find the equation of the tangent line to the curve when $t = -1$.

$$x(-1) = 11 \qquad y(-1) = -8$$

$$\frac{dx}{dt} = 3t^2 - 7 \qquad \left.\frac{dx}{dt}\right|_{t=-1} = -4$$

$$\frac{dy}{dt} = 6t^2 - 6t + 3 \qquad \left.\frac{dy}{dt}\right|_{t=-1} = 15$$

$$\left.\frac{dy}{dx}\right|_{t=-1} = -\frac{15}{4}$$

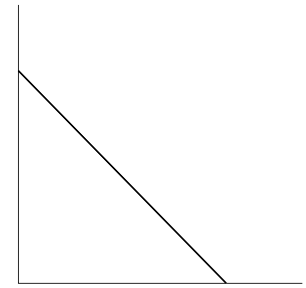
$$y + 8 = -\frac{15}{4}(x - 11)$$

(b) (6 points) Find all times t when the tangent line has slope 3.

From part (a) we have $\dfrac{dy}{dx} = \dfrac{6t^2 - 6t + 3}{3t^2 - 7}$.

$$\begin{aligned}
\frac{6t^2 - 6t + 3}{3t^2 - 7} &= 3 \\
6t^2 - 6t + 3 &= 9t^2 - 21 \\
0 &= 3t^2 + 6t - 24 \\
0 &= t^2 + 2t - 8 \\
0 &= (t-2)(t+4) \\
t &= -4, 2
\end{aligned}$$

$\boxed{3}$ (10 points) A ladder 13 ft long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 0.5 ft/s. Consider the triangle formed by the ladder, the ground and the wall. At what rate is the area of this triangle changing when the bottom of the ladder is 12 ft from the wall?



Let y be the height of the top of the ladder.

Let x be the distance of the bottom of the ladder from the wall.

Let A be the area of the triangle.

We know that $\dfrac{dx}{dt} = 0.5.$

We wish to compute $\dfrac{dA}{dt}$ *when* $x = 12.$

We have $A = \dfrac{1}{2}xy.$

By the Pythagorean Theorem, we have $y = \sqrt{169 - x^2}$

Thus $A = \dfrac{1}{2}x\sqrt{169 - x^2}.$

Differentiating with respect to t gives

$$\begin{aligned}
A &= \frac{1}{2}x\sqrt{169 - x^2} \\
\frac{dA}{dt} &= \frac{1}{2} \cdot \frac{dx}{dt} \cdot \sqrt{169 - x^2} + \frac{1}{2} \cdot x \cdot \frac{-2x}{2\sqrt{169 - x^2}} \cdot \frac{dx}{dt} \\
&= \frac{1}{2} \cdot 0.5 \cdot \sqrt{169 - 12^2} + \frac{1}{2} \cdot 12 \cdot \frac{-2 \cdot 12}{2\sqrt{169 - 12^2}} \cdot 0.5 \\
&= -\frac{119}{20} \\
&= -5.95 \ ft^2/sec
\end{aligned}$$

$\boxed{4}$ (8 points) Use a linear approximation to estimate $\sqrt[5]{33}$.

Let $f(x) = x^{1/5}$ *and take* $a = 32$.

$$\begin{aligned} f(32) &= 2 \\ f'(x) &= \frac{1}{5}x^{-4/5} \\ f'(32) &= \frac{1}{80} \\ L(x) &= 2 + \frac{1}{80}(x-32) \\ \sqrt[5]{33} &\approx L(33) \\ &= 2 + \frac{1}{80}(33-32) \\ &= \frac{161}{80} \end{aligned}$$

$\boxed{5}$ (8 points) Find all the points (a, b) on the curve $2x^3 + y^2 - 8y = 0$ where the tangent line is vertical.

$$\begin{aligned} 2x^3 + y^2 - 8y &= 0 \\ 6x^2 + 2yy' - 8y' &= 0 \\ y' &= -\frac{6x^2}{2y-8} \end{aligned}$$

Thus the tangent line is vertical when $y = 4$.

$$\begin{aligned} 2x^3 + y^2 - 8y &= 0 \\ 2x^3 + 4^2 - 8 \cdot 4 &= 0 \\ x^3 &= 8 \\ x &= 2 \end{aligned}$$

The only point on the curve with a vertical tangent is $(2, 4)$.