$\boxed{1}$ (6 points) Let $\phi(t) = \dfrac{4t+5}{11-3t}$. Find a formula for $\phi^{-1}(t)$.

$$\begin{aligned} y &= \frac{4t+5}{11-3t} \\ y(11-3t) &= 4t+5 \\ 11y - 3ty &= 4t+5 \\ 11y - 5 &= 3ty + 4t \\ 11y - 5 &= t(3y+4) \\ \frac{11y-5}{3y+4} &= t \end{aligned}$$

$\phi^{-1}(t) = \dfrac{11t-5}{3t+4}$.

$\boxed{2}$ (6 points) Let $f(x) = |x^2 - 4|$ and $g(x) = 2(x+5)$. Write out the multipart rule for the composition $f(g(x))$.

$f(x) = 4 - x^2$ if $-2 \le x \le 2$ and $f(x) = x^2 - 4$ otherwise.

Composition with $g(x)$ give a horizontal compression with a factor of $\frac{1}{2}$ followed by a horizontal translation 5 units to the left.

This shifts the interval $-2 \le x \le 2$ to the interval $-6 \le x \le -4$.

$$f(g(x)) = \begin{cases} [2(x+5)]^2 - 4 & \text{if } x < -6; \\ 4 - [2(x+5)]^2 & \text{if } -6 \le x \le -4; \\ [2(x+5)]^2 - 4 & \text{if } x > -4. \end{cases}$$

$\boxed{3}$ (13 points) Clovis is deciding how much to charge for his self-published novel. The number of copies he sells is a linear function of the amount that he charges.

If he charges \$15 per copy, he'll sell 350 copies.

If he charges \$30 per copy, he'll sell 230 copies.

(a) (6 points) Find a function $f(x)$ for the **total amount of money** Clovis earns by charging $\$x$ per copy.

Let y represent the number of copies he sells.
Then $\Delta y = -120$ and $\Delta x = 15$.
$\dfrac{\Delta y}{\Delta x} = -\dfrac{120}{15} = -8.$
The linear function is $y - 350 = -8(x - 15)$, or $y = -8x + 470$.

The total amount of money is the product of the price charged times the number of copies sold.

Thus $f(x) = x \cdot y = -8x^2 + 470x$.

(b) (7 points) How much should he charge in order to **maximize** his revenue?

We use the Vertex Formula $\dfrac{-b}{2a}$ to compute the value of x which maximizes the function $f(x)$.

Here, $b = 470$ and $a = -8$ so the x-coordinate of the vertex is $\dfrac{470}{2 \cdot 8}$.

Clovis should charge \$29.375 for a copy of his book.

$\boxed{4}$ (13 points) The population of Hawai'i was 1 million in 1980. It rose to 1.4 million in 2012. The population of Alaska was 0.5 million in 1984. It grew to 0.7 million in 2009.

(a) (4 points) Compute an exponential function that models the population of Hawai'i. Take $t = 0$ in 1980.

We want a function of the form $H(t) = A \cdot b^t$ *with* $H(0) = 1$ *and* $H(32) = 1.4$.

$1 = A \cdot b^0 = A$

$1.4 = A \cdot b^{32} \qquad b = (1.4)^{1/32} \approx 1.01057.$

$H(t) = (1.01057)^t.$

(b) (4 points) Compute an exponential function that models the population of Alaska. Take $t = 0$ in 1980.

We want a function of the form $A(t) = C \cdot d^t$ *with* $A(4) = 0.5$ *and* $A(29) = 0.7$.

$$\begin{aligned} 0.5 &= C \cdot d^4 \\ 0.7 &= C \cdot d^{29} \\ \frac{0.7}{0.5} &= \frac{C \cdot d^{29}}{C \cdot d^4} \\ 1.4 &= d^{25} \\ d &= (1.4)^{1/25} \approx 1.01355 \end{aligned}$$

Then $C = \dfrac{0.5}{d^4} \approx 0.4738$

$A(t) = 0.4738 \cdot (1.01355)^t.$

(c) (5 points) In what year will Hawai'i have twice as many people as Alaska?

We must solve the equation $H(t) = 2A(t)$ *for* t.

$$\begin{aligned} H(t) &= 2A(t) \\ (1.01057)^t &= 2 \cdot 0.4738 \cdot (1.01355)^t \\ \ln\left[(1.01057)^t\right] &= \ln\left[0.9476 \cdot (1.01355)^t\right] \\ \ln\left[(1.01057)^t\right] &= \ln(0.9476) + \ln\left[(1.01355)^t\right] \\ t \cdot \ln(1.01057) &= \ln(0.9476) + t \cdot \ln(1.01355) \\ t &= \frac{\ln(0.9476)}{\ln(1.01057) - \ln(1.01355)} \approx 18.28 \end{aligned}$$

The year is 1998.

$\boxed{5}$ (12 points) Find the linear-to-linear function whose graph passes through the points $(0, 1)$, $(1, 5)$ and $(2, 7)$. What is its horizontal asymptote?

Write $f(x) = \dfrac{ax + b}{x + c}$. *We must solve for* a, b *anc* c.

$$\begin{aligned} 1 &= f(0) \\ 1 &= \frac{b}{c} \\ c &= b \qquad (1) \end{aligned}$$

$$\begin{aligned} 5 &= f(1) \\ 5 &= \frac{a+b}{1+c} \\ 5 + 5c &= a + b \\ 5 + 5b &= a + b \qquad \textit{using (1) above} \\ 5 &= a - 4b \end{aligned}$$

$$\begin{aligned} 7 &= f(2) \\ 7 &= \frac{2a+b}{2+c} \\ 14 + 7c &= 2a + b \\ 14 + 7b &= 2a + b \qquad \textit{using (1) above} \\ 14 &= 2a - 6b \\ 7 &= a - 3b \end{aligned}$$

The linear system

$$\begin{aligned} 5 &= a - 4b \\ 7 &= a - 3b \end{aligned}$$

has solution $a = 13$ *and* $b = 2$. *By (1) we get* $c = 2$.

The function is $f(x) = \dfrac{13x + 2}{x + 2}$.

The horizontal asymptote is the line $y = 13$.