$\boxed{1}$ (12 points) Let $f(x) = x^2 - 5x$ and $g(x) = |3 - 2x|$

(a) (7 points) Simplify the expression $\dfrac{f(x+h) - f(x)}{h}$ far enough so that plugging in $h = 0$ would be allowed.

$$\begin{aligned}
f(x+h) &= (x+h)^2 - 5(x+h) \\
&= x^2 + 2xh + h^2 - 5x - 5h \\
\\
f(x+h) - f(x) &= (x^2 + 2xh + h^2 - 5x - 5h) - (x^2 - 5x) \\
&= 2xh + h^2 - 5h \\
\\
\frac{f(x+h) - f(x)}{h} &= 2x + h - 5
\end{aligned}$$

(b) (6 points) Find all solutions to the equation $g(x) = 3x - 7$.

*We must solve* $|3 - 2x| = 3x - 7$.
*This gives two equations:*
*1.* $3 - 2x = 3x - 7$. *This has solution* $x = 2$. *But plugging* $x = 2$ *back into the original equation gives* $1 = -1$. *This equation does not give a solution.*

*2.* $-(3 - 2x) = 3x - 7$. *This has solution* $x = 4$. *Plugging* $x = 4$ *back into the original equation gives* $5 = 5$. *This equation does give a solution.*
*The only solution is* $x = 4$.

$\boxed{2}$ (13 points) Clovis and Isobel are standing on Broadway, 30 feet South of the intersection with Aloha St. Clovis starts walking North at a constant rate of 5 feet/second. When he reaches the intersection, he turns West and continues at the same speed down Aloha St. Isobel does not move.

(a) (7 points) Give a multi-part function for the distance between Clovis and Isobel as a function of time. Use units of feet and seconds.

*Clovis is traveling in a straight line away from Isobel for the first 6 seconds. After that he is traveling at right angles to her and we must use the distance formula.*

$$d(t) = \begin{cases} 5t & \text{if } 0 \le t \le 6; \\ \sqrt{900 + 25(t-6)^2} & \text{if } t > 6. \end{cases}$$

(b) (6 points) When are they 50 feet apart?

*Since the intersection is only 30 feet away, we can assume $t > 6$.*

$$\begin{aligned} 50 &= \sqrt{900 + 25(t-6)^2} \\ 2500 &= 900 + 25(t-6)^2 \\ 1600 &= 25(t-6)^2 \\ 64 &= (t-6)^2 \\ \pm 8 &= t - 6 \\ t &= -2, 14 \end{aligned}$$

*The only answer that makes sense is $t = 14$ seconds.*

$\boxed{3}$ (12 points) Tafu is sailing near a radar buoy which can detect anything within 9 km of the buoy. He starts sailing from a point 7 km West and 11 km North of the buoy. He sails South for one hour, then turns and sails East for 30 km.

He sails at a constant speed of 6 km/hr.

How much time was he within 9 km of the buoy?

*Introduce coordinates with the buoy at the origin, the $x$-axis going East-West and the $y$-axis going North-South.*

*Then the circle around the buoy has equation* $x^2 + y^2 = 81$

*Tafu is initially traveling South along the line* $x = -7$.

*We calculate the intersection of the line and the circle:*

$$\begin{aligned} (-7)^2 + y^2 &= 81 \\ y^2 &= 32 \\ y &= \pm 4\sqrt{2} \end{aligned}$$

*Thus Tafu enters the circle at the point* $(-7, 4\sqrt{2})$.

*Traveling South for one hour from his initial position at 6 km/hr takes him to the point* $(-7, 5)$.

*His total distance in the circle on his Southward voyage is* $4\sqrt{2} - 5$ *km.*

*This part of his voyage in the circle takes* $T_1 = \dfrac{4\sqrt{2} - 5}{6} \approx 0.11$ *hours.*

*Now Tafu is travelling East along the line* $y = 5$ *and he starts at* $(-7, 5)$.

*Traveling 30 km will take him out of the circle.*

*We intersect the line and the circle:*

$$\begin{aligned} x^2 + (5)^2 &= 81 \\ x^2 &= 56 \\ x &= \pm\sqrt{56} \end{aligned}$$

*Thus Tafu exits the circle at the point* $(\sqrt{56}, 5)$.

*His total distance in the circle on his Eastward voyage is* $\sqrt{56} - (-7)$ *km.*

*This part of his voyage in the circle takes* $T_2 = \dfrac{\sqrt{56} + 7}{6} \approx 2.41$ *hours.*

*The total time is* $T_1 + T_2 = \dfrac{4\sqrt{2} + \sqrt{56} + 2}{6} \approx 2.52$ *hours.*

$\boxed{4}$ (12 points) Winfield is moving linearly in the $xy$-plane at a constant speed. He starts from the point $(3, -1)$ and moves along the line $y = -2x + 5$ at a speed of 3 units per second, heading toward the $y$-axis.

(a) (6 points) Write parametric equations for Winfield's location $t$ seconds after starting.

*Win starts at* $(3, -1)$ *and goes to the point* $(0, 5)$*, the* $y$*-intercept of the line.*
*The distance is* $\sqrt{3^2 + 6^2} = 3\sqrt{5}$ *units.*
*He has a speed of 3 units/second so we compute* $\Delta t = \dfrac{3\sqrt{5}}{3} = \sqrt{5}$ *seconds.*
*Now plug into* $x = v_x t + x_0$, $y = v_y t + y_0$.

$(x_0, y_0) = (3, -1)$

$$v_x = \frac{\Delta x}{\Delta t} = \frac{0 - 3}{\sqrt{5}}$$

$$v_x = \frac{\Delta y}{\Delta t} = \frac{5 - (-1)}{\sqrt{5}}$$

*The equations are*

$$x = -\frac{3}{\sqrt{5}}t + 3, \;\; y = \frac{6}{\sqrt{5}}t - 1$$

(b) (6 points) At what time is Winfield closest to the origin?

*The line through the origin perpendicular to Win's path is* $y = \dfrac{1}{2}x$.
*We find the* $x$*-coordinate of the intersection of the two lines:*

$$\begin{aligned} \frac{1}{2}x &= -2x + 5 \\ x &= -4x + 10 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

*We use the* $x$*-parametric equation to solve for* $t$*:*

$$\begin{aligned} 2 &= -\frac{3}{\sqrt{5}}t + 3 \\ \frac{3}{\sqrt{5}}t &= 1 \\ t &= \frac{\sqrt{5}}{3} \end{aligned}$$

*The answer is* $t = \dfrac{\sqrt{5}}{3} \approx 0.745$ *seconds.*