Let \( f(x) = x^2 - 5x \) and \( g(x) = |3 - 2x| \)

(a) (7 points) Simplify the expression \( \frac{f(x + h) - f(x)}{h} \) far enough so that plugging in \( h = 0 \) would be allowed.

\[
f(x + h) = (x + h)^2 - 5(x + h) \\
= x^2 + 2xh + h^2 - 5x - 5h
\]

\[
f(x + h) - f(x) = (x^2 + 2xh + h^2 - 5x - 5h) - (x^2 - 5x) \\
= 2xh + h^2 - 5h
\]

\[
\frac{f(x + h) - f(x)}{h} = 2x + h - 5
\]

(b) (6 points) Find all solutions to the equation \( g(x) = 3x - 7 \).

We must solve \( |3 - 2x| = 3x - 7 \).

This gives two equations:
1. \( 3 - 2x = 3x - 7 \). This has solution \( x = 2 \). But plugging \( x = 2 \) back into the original equation gives \( 1 = -1 \). This equation does not give a solution.

2. \( -(3 - 2x) = 3x - 7 \). This has solution \( x = 4 \). Plugging \( x = 4 \) back into the original equation gives \( 5 = 5 \). This equation does give a solution.

The only solution is \( x = 4 \).
(13 points) Clovis and Isobel are standing on Broadway, 30 feet South of the intersection with Aloha St. Clovis starts walking North at a constant rate of 5 feet/second. When he reaches the intersection, he turns West and continues at the same speed down Aloha St. Isobel does not move.

(a) (7 points) Give a multi-part function for the distance between Clovis and Isobel as a function of time. Use units of feet and seconds.

Clovis is traveling in a straight line away from Isobel for the first 6 seconds. After that he is traveling at right angles to her and we must use the distance formula.

\[
d(t) = \begin{cases} 
5t & \text{if } 0 \leq t \leq 6; \\
\sqrt{900 + 25(t - 6)^2} & \text{if } t > 6. 
\end{cases}
\]

(b) (6 points) When are they 50 feet apart?

Since the intersection is only 30 feet away, we can assume \( t > 6 \).

\[
\begin{align*}
50 &= \sqrt{900 + 25(t - 6)^2} \\
2500 &= 900 + 25(t - 6)^2 \\
1600 &= 25(t - 6)^2 \\
64 &= (t - 6)^2 \\
\pm 8 &= t - 6 \\
t &= -2, 14
\end{align*}
\]

The only answer that makes sense is \( t = 14 \) seconds.
Tafu is sailing near a radar buoy which can detect anything within 9 km of the buoy. He starts sailing from a point 7 km West and 11 km North of the buoy. He sails South for one hour, then turns and sails East for 30 km. He sails at a constant speed of 6 km/hr.

How much time was he within 9 km of the buoy?

Introduce coordinates with the buoy at the origin, the x-axis going East-West and the y-axis going North-South.

Then the circle around the buoy has equation \( x^2 + y^2 = 81 \)

Tafu is initially traveling South along the line \( x = -7 \).

We calculate the intersection of the line and the circle:

\[
(-7)^2 + y^2 = 81 \\
y^2 = 32 \\
y = \pm4\sqrt{2}
\]

Thus Tafu enters the circle at the point \((-7, 4\sqrt{2})\).

Traveling South for one hour from his initial position at 6 km/hr takes him to the point \((-7, 5)\).

His total distance in the circle on his Southward voyage is \( 4\sqrt{2} - 5 \) km.

This part of his voyage in the circle takes \( T_1 = \frac{4\sqrt{2} - 5}{6} \approx 0.11 \) hours.

Now Tafu is travelling East along the line \( y = 5 \) and he starts at \((-7, 5)\).

Traveling 30 km will take him out of the circle.

We intersect the line and the circle:

\[
x^2 + (5)^2 = 81 \\
x^2 = 56 \\
x = \pm\sqrt{56}
\]

Thus Tafu exits the circle at the point \((\sqrt{56}, 5)\).

His total distance in the circle on his Eastward voyage is \( \sqrt{56} - (-7) \) km.

This part of his voyage in the circle takes \( T_2 = \frac{\sqrt{56} + 7}{6} \approx 2.41 \) hours.

The total time is \( T_1 + T_2 = \frac{4\sqrt{2} + \sqrt{56} + 2}{6} \approx 2.52 \) hours.
Winfield is moving linearly in the $xy$-plane at a constant speed. He starts from the point $(3, -1)$ and moves along the line $y = -2x + 5$ at a speed of 3 units per second, heading toward the $y$-axis.

(a) (6 points) Write parametric equations for Winfield’s location $t$ seconds after starting.

Win starts at $(3, -1)$ and goes to the point $(0, 5)$, the $y$-intercept of the line. The distance is $\sqrt{3^2 + 6^2} = 3\sqrt{5}$ units.

He has a speed of 3 units/second so we compute $\Delta t = \frac{3\sqrt{5}}{3} = \sqrt{5}$ seconds.

Now plug into $x = v_x t + x_0, y = v_y t + y_0$.

$(x_0, y_0) = (3, -1)$

$v_x = \frac{\Delta x}{\Delta t} = \frac{0 - 3}{\sqrt{5}}$

$v_y = \frac{\Delta y}{\Delta t} = \frac{5 - (-1)}{\sqrt{5}}$

The equations are

$x = -\frac{3}{\sqrt{5}} t + 3, \ y = \frac{6}{\sqrt{5}} t - 1$

(b) (6 points) At what time is Winfield closest to the origin?

The line through the origin perpendicular to Win’s path is $y = \frac{1}{2} x$.

We find the $x$-coordinate of the intersection of the two lines:

\[
\frac{1}{2} x = -2x + 5 \\
x = -4x + 10 \\
5x = 10 \\
x = 2
\]

We use the $x$-parametric equation to solve for $t$:

\[
2 = -\frac{3}{\sqrt{5}} t + 3 \\
\frac{3}{\sqrt{5}} t = 1 \\
t = \frac{\sqrt{5}}{3}
\]

The answer is $t = \frac{\sqrt{5}}{3} \approx 0.745$ seconds.