

## Math 112

### Solutions for Group Activity: Multivariable Functions

So far, in Math 111/112, we've investigated functions that have only one input variable, like  $TR(q) = 25q - 0.5q^2$ , which has one input variable, quantity  $q$ . For the remainder of the quarter, we'll study functions with more than one input variable. These are known as **multivariable functions**.

1. The balance in a savings account with continuously-compounded interest is given by the formula

$$A(P, r, t) = Pe^{rt},$$

where  $P$  is the principal (the amount initially invested),  $r$  is the annual interest rate expressed as a decimal, and  $t$  is time in years that the account has been accruing interest.

- (a) Compute  $A(5000, 0.06, 8)$  and write a sentence or two describing what it represents.

**ANSWER:**  $A(5000, 0.06, 8) = 5000e^{0.06(8)} = \$8080.37$ . This is the balance in the account after 8 years if \$5000 is invested at an annual rate of 6%.

- (b) Suppose you have exactly \$10,000 to use as principal and the only account available pays 4% interest, compounded continuously. Then the only variable that can change is time  $t$ .

For each of the following, translate into functional notation and compute.

- i. the change in the balance from  $t = 4$  to  $t = 9$  years

**ANSWER:**  $A(10000, 0.04, 9) - A(10000, 0.04, 4) = \$2598.19$

- ii. the average rate at which the balance changes (in dollars per year) from  $t = 4$  to  $t = 9$  years

**ANSWER:**  $\frac{A(10000, 0.04, 9) - A(10000, 0.04, 4)}{5} = \frac{\$2598.19}{5} = \$519.64$  per year

- (c) Suppose you've found an investment that promises 5% annual interest, compounded continuously, for a term of exactly 10 years. Then the only variable that can change is the principal  $P$ .

- i. You have \$1000 of your own to invest. Your friend offers to give you another \$500. How much would adding your friend's \$500 to the principal increase the pay-off amount of the investment?

**ANSWER:** The change in the pay-off amount is  $A(1500, 0.05, 10) - A(1000, 0.05, 10) = \$824.36$

- ii. By how much will the pay-off amount increase if you increase the principal by one dollar: from  $P$  to  $P + 1$ ?

**SOLUTION:** With a principal of  $\$P$ , the balance after ten years is  $A(P, 0.05, 10) = Pe^{0.05(10)}$ . With a principal of  $\$(P + 1)$ , the balance after ten years is  $A(P + 1, 0.05, 10) = (P + 1)e^{0.05(10)}$ . The change in the pay-off amount is then:

$$A(P + 1, 0.05, 10) - A(P, 0.05, 10) = (P + 1)e^{0.5} - Pe^{0.5} = Pe^{0.5} + e^{0.5} - Pe^{0.5} = e^{0.5} = \$1.65.$$

2. In a certain math course, the final grade is determined by computing a weighted average of homework, participation, two midterm exams, and a final exam. The total number of points available for each component and its weighting is given in the following table.

| Component     | Points Earned | Points Possible | Weighting |
|---------------|---------------|-----------------|-----------|
| Homework      | $h$           | 600             | 15%       |
| Participation | $p$           | 16              | 5%        |
| Exam I        | $x$           | 50              | 22%       |
| Exam II       | $y$           | 50              | 22%       |
| Final         | $z$           | 100             | 36%       |

At the end of the quarter, a student's total percentage is given by:

$$C(h, p, x, y, z) = \left(\frac{h}{600}\right) 15 + \left(\frac{p}{16}\right) 5 + \left(\frac{x}{50}\right) 22 + \left(\frac{y}{50}\right) 22 + \left(\frac{z}{100}\right) 36,$$

which simplifies to

$$C(h, p, x, y, z) = 0.025h + 0.3125p + 0.44x + 0.44y + 0.36z.$$

This percentage is then converted into a grade as follows:

- If  $C \geq 97$ , then the student receives a 4.0 in the course.
- If  $94 \leq C \leq 96$ , then the student receives a 3.9 in the course.
- If  $70 \leq C \leq 92$ , then the student's grade is  $0.1C - 5.5$ .

- (a) Terry earns 567 homework points, has a perfect participation score, and scores 43, 39, and 85 on the exams.

- i. Compute Terry's total percentage:  $C(567, 16, 43, 39, 85)$ . (Round to the nearest whole number.)

**ANSWER:**  $C(567, 16, 43, 39, 85) = 86$

- ii. What grade does Terry receive in the course?

**ANSWER:** Terry earns a 3.1 in the course.

- (b) Chris needs to earn at least a 2.7 in the course to keep a scholarship.

- i. What total percentage  $C$  must Chris earn to receive a 2.7 in the course?

**SOLUTION:** Set  $0.1C - 5.5$  equal to 2.7 and solve for  $C$ .

**ANSWER:** Chris needs an 82% in the course, overall.

- ii. Before the final, Chris has earned 576 homework points, 15 participation points, and midterm scores of 41 and 40. What must Chris earn on the final in order to receive a 2.7 in the course? (Round to the nearest whole number.)

**SOLUTION:** Set  $C(576, 15, 41, 40, z) = 82$  and solve for  $z$ .

**ANSWER:** Chris needs to earn a 76 on the final exam.

- (c) Pat requests a regrade on Exam II and receives 2 additional points on that exam. If no other scores change, how much will Pat's total percentage  $C$  increase?

**SOLUTION:**  $C(h, p, x, y + 2, z) - C(h, p, x, y, z) = (0.025h + 0.3125p + 0.44x + 0.44(y + 2) + 0.36z) - (0.025h + 0.3125p + 0.44x + 0.44y + 0.36z) = 0.44(2) = 0.88.$

**ANSWER:** Pat's overall percentage will increase by 0.88 points.

- (d) Which will lead to the largest increase in a student's total percentage: a 50-point increase in homework or a 5-point increase on the final exam?

**SOLUTION:**  $C(h + 50, p, x, y, z) - C(h, p, x, y, z) = (0.025(h + 50) + 0.3125p + 0.44x + 0.44y + 0.36z) - (0.025h + 0.3125p + 0.44x + 0.44y + 0.36z) = 0.025(50) = 1.25.$

$C(h, p, x, y, z + 5) - C(h, p, x, y, z) = (0.025h + 0.3125p + 0.44x + 0.44y + 0.36(z + 5)) - (0.025h + 0.3125p + 0.44x + 0.44y + 0.36z) = 0.36(5) = 1.8.$

**ANSWER:** A five-point increase in the final will lead to a larger increase in total percentage than a fifty-point increase on homework. (**Editorial Note:** But doing the homework will lead to a higher final exam score.)