Math 112

Group Activity: Total Revenue and Total Cost from Marginal Revenue and Marginal Cost

Recall:

- The graphs of total revenue and variable cost go through the origin:
  - $TR(0) = 0$
  - $VC(0) = 0$
- The "y"-intercept of total cost is fixed cost: $TC(0) = FC$.
- Total cost is the sum of variable cost and fixed cost, which means that the graph of total cost is a vertical shift of the graph of variable cost.
$$TC(q) = VC(q) + FC.$$
- The derivative of total revenue is marginal revenue: $MR(q) = TR'(q)$.
- The derivative of total cost and the derivative of variable cost are the same. Both are equal to marginal cost:
$$TC'(q) = VC'(q) = MC(q).$$

1. (a) Find $MR(q)$ if $TR(q) = -\frac{3}{2}q^2 + 20q$.

   ANSWER: $\boxed{MR(q) = -3q + 20}$

   (b) Find $TR(q)$ if $MR(q) = 100 - 9q$.

   ANSWER: $\boxed{TR(q) = 100q - \frac{9}{2}q^2}$

   (c) Find FC, $VC(q)$, and $MC(q)$ if $TC(q) = \sqrt{q + 64}$.

   ANSWER: $\boxed{FC = TC(0) = 8}$

   $\boxed{VC(q) = TC(q) - FC = \sqrt{q + 64} - 8}$

   $\boxed{MC(q) = TC'(q) = VC'(q) = \frac{1}{2}(q + 64)^{-1/2}}$

   (d) Find $VC(q)$ and $TC(q)$ if $MC(q) = 30\sqrt{q + 100}$ and $FC = 50,000$.

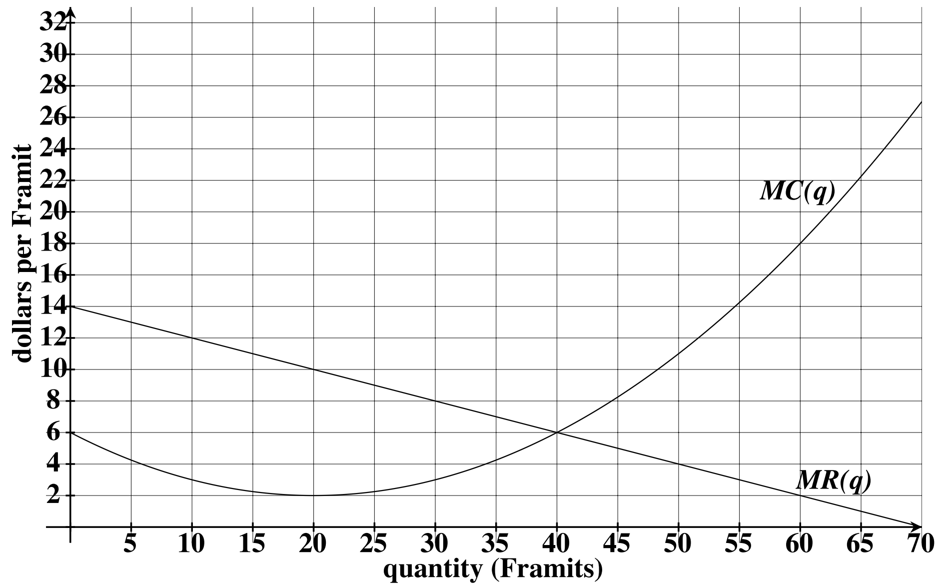
   SOLUTION: Both VC and TC are anti-derivatives of $MC(q)$. The general anti-derivative of $MC(q)$ is
$$\int 30\sqrt{q + 100}\, dq = \int 30(q + 100)^{1/2}\, dq = 30 \cdot \frac{2}{3}(q + 100)^{3/2} + K = 20(q + 100)^{3/2} + K.$$

   So, $TC(q) = 20(q + 100)^{3/2} + K$, for some particular constant K. To find K, we use the fact that $TC(0) = FC = 50,000$:
$$TC(0) = 20(0 + 100)^{3/2} + K = 20,000 + K = 50,000,$$

   which means that $K = 30,000$. So, $TC(q) = 20(q + 100)^{3/2} + 30,000$. Then $VC(q) = TC(q) - FC$.

   ANSWER: $\boxed{TC(q) = 20(q + 100)^{3/2} + 30,000 \text{ and } VC(q) = 20(q + 100)^{3/2} - 20,000.}$

2. The graph below shows the graphs of marginal revenue and marginal cost to sell and produce Framits.



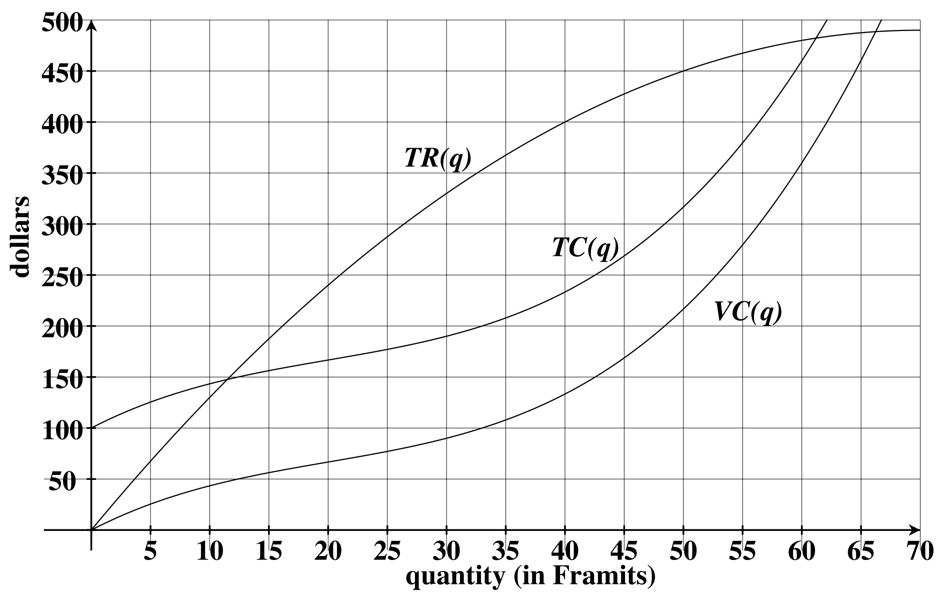
(a) Define a function

$$A(q) = \text{the area under } MR \text{ from 0 to } q.$$

Fill in the values in the following table:

q	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70
$A(q)$	0	67.5	130	187.5	240	287.5	330	367.5	400	427.5	450	467.5	480	487.5	490

(b) Sketch the graph of $A(q)$ on the following set of axes:



We've learned in previous activities that such an area function gives us an anti-derivative of $MR(q)$. Moreover, this is the anti-derivative of $MR(q)$ that goes through the origin. Thus, you have just sketched the graph of total revenue. Label this graph $TR(q)$.

FACT: The area under MR from 0 to q always gives $TR(q)$.

Getting from MC to TC will be harder. For one thing, in this scenario, the graph of MR is a line—we can easily compute, for example, the area under MR from 0 to 45 as the area of a *single* trapezoid. To compute the area under MC, however, we will have to break the region up into smaller pieces that approximate trapezoids. Further, the area under MC will give *an* anti-derivative of MC—we'll need to consider how VC and FC fit into this picture.

(c) Fill in the following table with the area under the MC graph on the indicated interval.

Interval	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45–50	50–55	55–60	60–65	65–70
Area under MC	25.5	18	13	10.5	10.5	13	18	25.5	35.5	48	63	80.5	100.5	123

(d) Add together the appropriate areas from part (c) to fill in the following table with the area under the MC graph from 0 to q.

q	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70
area under MC from 0 to q	0	25.5	43.5	56.5	67	77.5	90.5	108.5	134	169.5	217.5	280.5	361	461.5	584

(e) We know that the table in part (d) gives values of a function that is an anti-derivative of MC. Moreover, this function goes through the origin. The anti-derivative of MC that goes through the origin is variable cost. On the axes in part (b), sketch and label the graph of $VC(q)$.

FACT: The area under MC from 0 to q always gives $VC(q)$.

(f) Fixed costs are \$100. Sketch and label the graph of $TC(q)$ on the axes in part (b).

(g) What quantity gives the largest possible profit?

SOLUTION: See where the graphs of MR and MC intersect.

ANSWER: Profit is maximized at $q = 40$ Framits.

(h) What is the largest possible profit?

SOLUTION: $TR(40) = 400$, $VC(40) = 134$, and $TC(40) = 234$.

ANSWER: Maximum profit is \$166.

(i) What is the largest quantity at which you won't be forced to take a loss?

SOLUTION: On the graphs of TR and TC, find the largest quantity at which $TR = TC$. For larger quantities, TC will be larger than TR.

ANSWER: $q \approx 61$ Framits