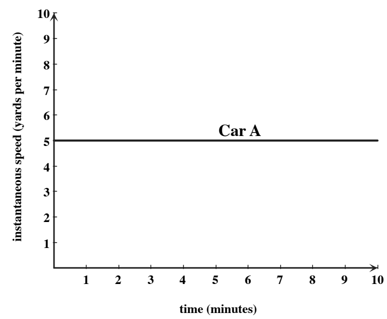
**Math 112**

Solutions for Group Activity: Distance Traveled from Speed

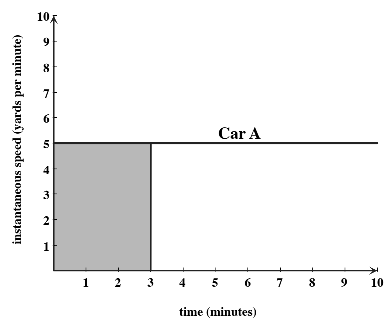
**Recall:** Instantaneous speed is the derivative of distance traveled. Given a formula for distance traveled, $D(t)$, we simply compute the derivative $D'(t)$ to obtain instantaneous speed at time $t$. Given the graph of distance traveled, to compute instantaneous speed at time $t$, we find the slope of the line tangent to distance traveled at that time. The slope of a tangent line to the graph of distance traveled becomes a "$y$"-value on the graph of instantaneous speed.

**Goal:** To investigate how we find values of distance traveled given the graph of instantaneous speed.

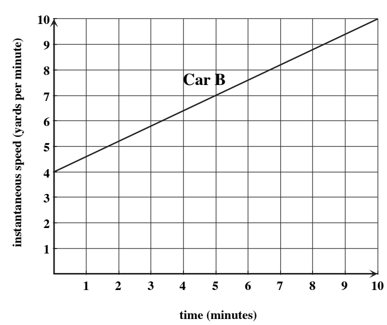
1. The graph at right shows the instantaneous speed of Car $A$. Car $A$ travels at a constant speed of 5 yards per minute. Compute the distance Car $A$ travels during the first three minutes. (How far does it go if it travels at a constant speed of 5 yards per minute for 3 minutes?)
   **ANSWER:** 5 yards per minute for 3 minutes gives a distance traveled of 15 yards.




2. Compute the *area* under Car $A$'s speed graph and above the $t$-axis from $t = 0$ to $t = 3$. What do you notice?
   **ANSWER:** The area is 15, the same as distance traveled from $t = 0$ to $t = 3$.




3. The graph at right shows the instantaneous speed of Car $B$. Car $B$ does not travel at a constant speed—it is getting faster all the time. How fast is Car $B$ traveling at $t = 0$? At $t = 5$?
   **ANSWER:** At $t = 0$, Car $B$ is traveling at a rate of 4 yards per minute. At $t = 5$, it's traveling at a rate of 10 yards per minute.




4. The following is always true: if an object travels with a linear speed, its average speed during that time interval is exactly half-way between its highest and lowest speeds on that interval. Use this fact and your answer to the previous question to determine the *average speed* of Car $B$ on the interval from $t = 0$ to $t = 5$.

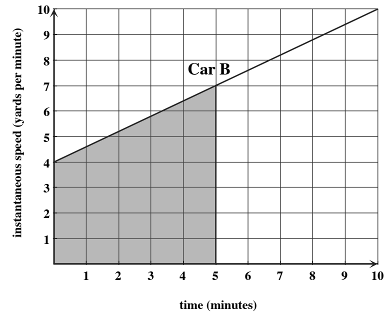
   **ANSWER:** The average speed of Car $B$ on the interval from $t = 0$ to $t = 5$ is half-way between 4 and 7 yards per minute: 5.5 yards per minute.

5. We know that average speed $= \dfrac{\text{distance traveled}}{\text{time elapsed}}$. Use this fact and your answer to the previous question to compute the distance Car $B$ travels from $t = 0$ to $t = 5$.

   **SOLUTION:** Distance traveled = (average speed)$\times$(time elapsed). On the interval from $t = 0$ to $t = 5$, average speed is 5.5 yards per minute and time elapsed is 5 minutes.

   **ANSWER:** Distance traveled is 5.5 yards per min $\times$ 5 min = 27.5 yards.

6. Compute the *area* under Car $B$'s speed graph and above the $t$-axis from $t = 0$ to $t = 5$. (It may help to view this region as a triangle on top of a rectangle.) What do you notice?
**ANSWER:** The area is 27.5, the same as distance traveled.



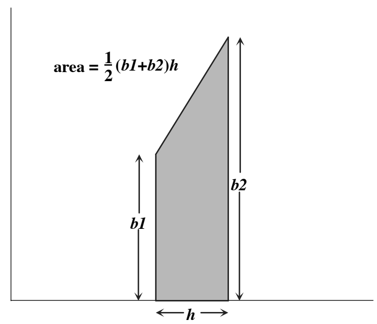

In general, given the graph of instantaneous speed, distance traveled from $t = a$ to $t = b$ is equal to the *area* of the region under the speed graph and above the $t$-axis from $t = a$ to $t = b$.

7. Use the graph above to compute the distance Car $B$ travels from $t = 5$ to $t = 10$. (Again, view the region whose area you're computing as a triangle on top of a rectangle.)

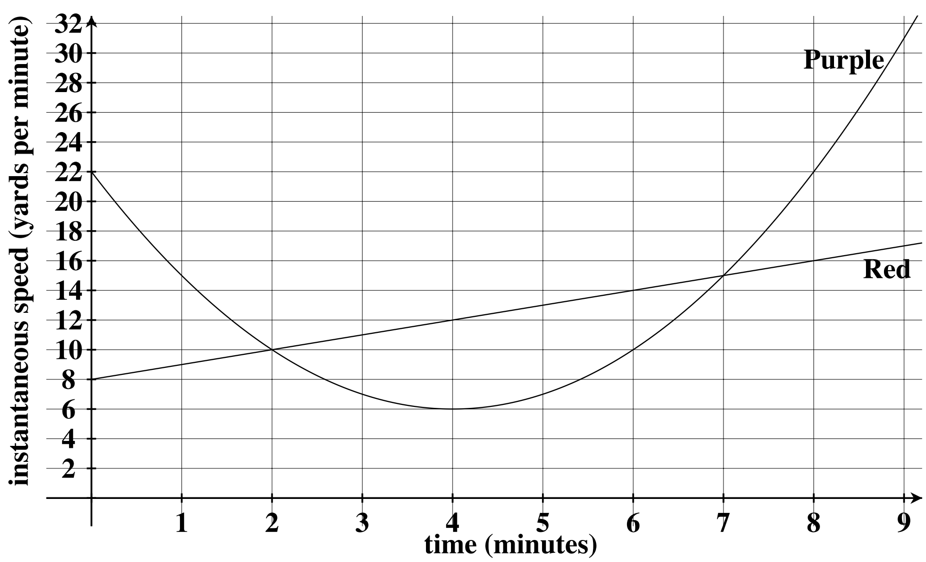
**SOLUTION:** Compute the area under the speed graph from $t = 5$ to $t = 10$.

**ANSWER:** 42.5 yards

For the rest of this activity, you'll be computing a lot of areas. Most of the regions whose areas you'll need will be trapezoids. The graph at right shows a quick formula for computing the area of a trapezoid of the type you will see.




8. A red car and a purple car travel along a straight track. At $t = 0$, both cars are next to each other. The graph below shows their instantaneous speeds. We'll use areas to find the graph of distance traveled for each car.




(a) On the interval from $t = 0$ to $t = 1$, the graph of the Red car's speed is linear. Compute the area under the Red car's speed graph on this interval to find the distance Red travels from $t = 0$ to $t = 1$.
**ANSWER:** 8.5 yards

(b) The Purple car's speed graph is *not* linear. However, on a short enough interval, a curvy graph looks *almost* linear. In particular, on the interval from $t = 0$ to $t = 1$, the area under the Purple car's speed graph can be approximated by the area of a trapezoid. Compute this area to estimate the distance Purple travels in the first minute.

**ANSWER:** 18.5 yards

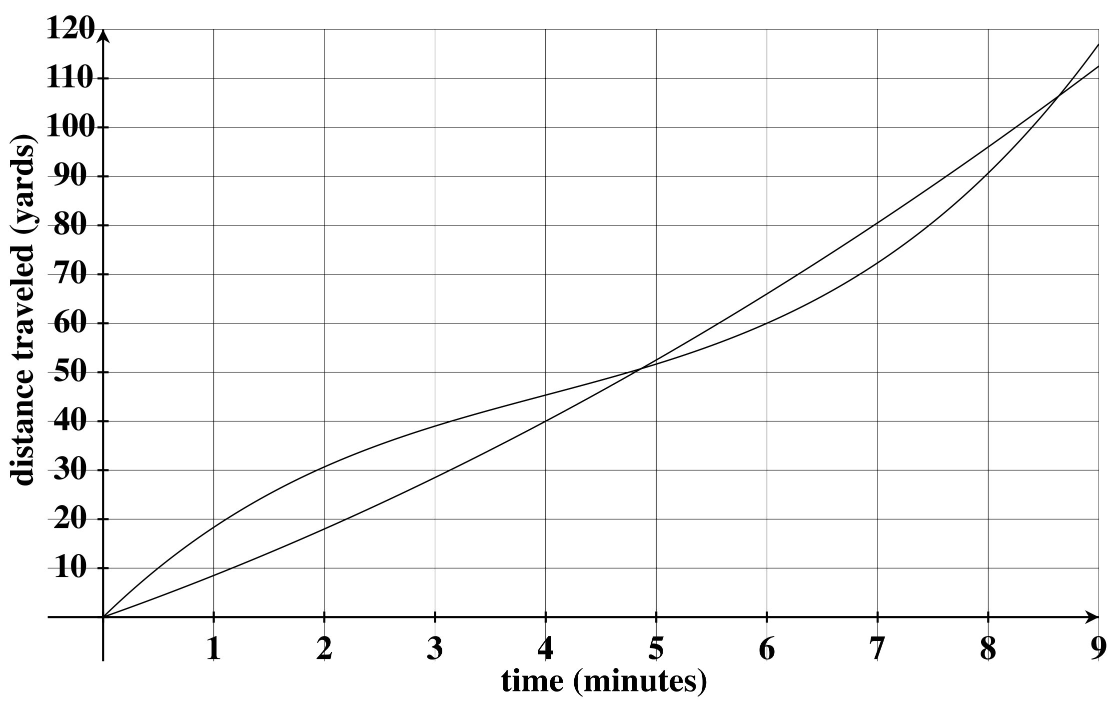
(c) Check your answers to parts (a) and (b) against the first entry for each car in the following table and compute the appropriate areas to fill in the remaining entries.

| Interval | 0–1 | 1–2 | 2–3 | 3–4 | 4–5 | 5–6 | 6–7 | 7–8 | 8–9 |
|---|---|---|---|---|---|---|---|---|---|
| Distance covered by Red car | 8.5 | 9.5 | 10.5 | 11.5 | 12.5 | 13.5 | 14.5 | 15.5 | 16.5 |
| Distance covered by Purple car | 18.5 | 12.5 | 8.5 | 6.5 | 6.5 | 8.5 | 12.5 | 18.5 | 26.5 |

(d) The table in the previous question gives us distance traveled by each car over one-minute intervals. To get distance traveled over wider intervals, we can add these areas together. For example, in the interval from $t = 0$ to $t = 3$, the Purple car travels approximately $18.5 + 12.5 + 8.5 = 39.5$ yards. Add the appropriate areas to fill in the following table, giving distance traveled from time 0 to time $t$.

| time $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Distance covered by Red car from 0 to $t$ | 0 | 8.5 | 18 | 28.5 | 40 | 52.5 | 66 | 80.5 | 96 | 112.5 |
| Distance covered by Purple car from 0 to $t$ | 0 | 18.5 | 31 | 39.5 | 46 | 52.5 | 61 | 73.5 | 92 | 118.5 |

(e) Sketch the graphs of distance traveled for the Red car and the Purple car on the axes below.




(f) Use the distance graphs to estimate the times at which the two cars are farthest apart. What's true about the speed graphs at those times?

**ANSWER:** The distance graphs look farthest apart at $t = 2$ and $t = 7$. These are the times when the graphs of instantaneous speed cross.