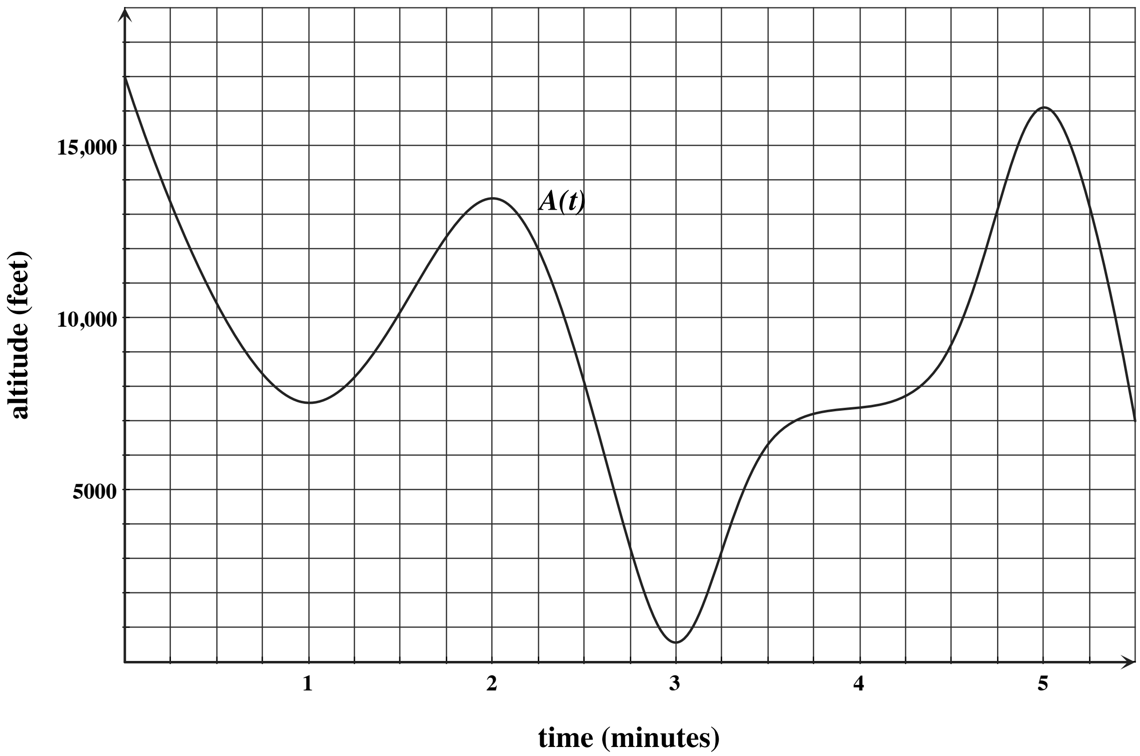
Name: ______________________________ Section: __________

Math 112

Solutions for Group Activity: Local vs. Global Optima

1. Below is the graph of altitude, $A(t)$, for a balloon that is rising and falling.



(a) List all critical values of the function $A(t)$.

ANSWER: The critical values of $A(t)$ are $t = 1, 2, 3, 4,$ and 5.

(b) Which of these give local (relative) maxima? local minima? horizontal points of inflection?

ANSWER: $A(t)$ has relative maxima at $t = 2$ and $t = 5$, relative minima at $t = 1$ and $t = 3$, and a horizontal point of inflection at $t = 4$.

(c) On the interval from $t = 0$ to $t = 5.5$, the balloon is at its highest altitude at $t = 0$. We say that the **global maximum** of $A(t)$ *occurs at* $t = 0$. What is that altitude?

ANSWER: The highest altitude the balloon reaches is 17,000 feet.

(d) At what time does the **global minimum** of $A(t)$ (the *lowest* altitude) occur? What is the lowest altitude the balloon reaches on the interval shown?

ANSWER: The balloon is at its lowest altitude at $t = 3$ minutes. The lowest altitude the balloon reaches is approximately 600 feet.

(e) We're now going to restrict our attention to smaller time intervals. Fill in the following chart with the values of t_{max}, the value of t that gives the global maximum altitude on the given interval, and t_{min}, the value of t that gives the global minimum altitude on the given interval.

Interval	t_{max}	t_{min}
from $t = 0$ to $t = 5.5$	0	3
from $t = 0.75$ to $t = 2.25$	1	2
from $t = 1.75$ to $t = 2.75$	2	2.75
from $t = 4.5$ to $t = 4.75$	4.5	4.75

The previous exercise demonstrates the following fact: On the interval from $x = a$ to $x = b$, the global maximum value of a function $f(x)$ occurs either at a critical value or at one of the endpoints of the interval. That is, the global maximum value of $f(x)$ is either a local maximum value of $f(x)$ OR it is $f(a)$ or $f(b)$. Similarly, the global minimum value of $f(x)$ is either a local minimum value of $f(x)$ OR it is $f(a)$ or $f(b)$.

This gives a convenient process to find the local and global maximum and minimum vlaues of a function $f(x)$ on the interval from $x = a$ to $x = b$.

Step 1: Compute $f'(x)$, set $f'(x) = 0$, and solve for x. This gives the critical values for $f(x)$.

Step 2: Evaluate $f(x)$ at each of the critical values **that lie between** a **and** b (you can ignore any critical values that are not in your interval). Also, compute $f(a)$ and $f(b)$.

Step 3: Use the information gathered in Steps 1 and 2 to SKETCH A ROUGH GRAPH of $f(x)$ on the interval from a to b. You should be able to see on your graph all local and global optima.

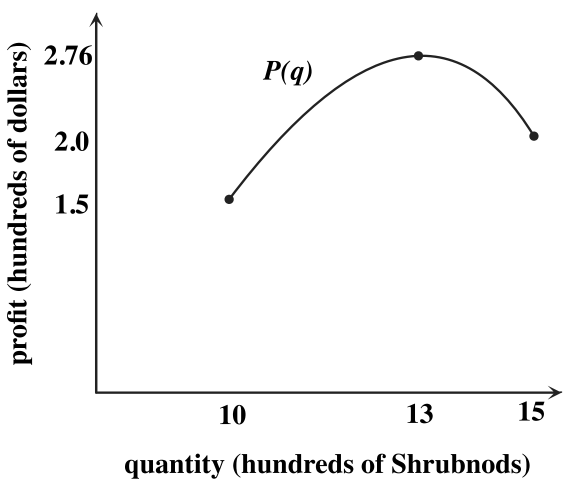
2. The total revenue and total cost (both in hundreds of dollars) for selling q hundred Shrubnods are given by:

$$TR(q) = -0.08q^2 + 2.35q \text{ and } TC(q) = 0.01q^3 - 0.3q^2 + 3q + 4.$$

(a) Compute the formula for profit $P(q)$ and its derivative $P'(q)$.
ANSWER: $P(q) = -0.01q^3 + 0.22q^2 - 0.65q - 4$ and $P'(q) = -0.03q^2 + 0.44q - 0.65$

(b) Find all critical values of the profit function. (You may round your answers to 2 digits after the decimal.)
SOLUTION: Set $P'(q) = 0$ and solve for q using the quadratic formula.
ANSWER: The critical values of profit are $q = 1.67$ and $q = 13$.

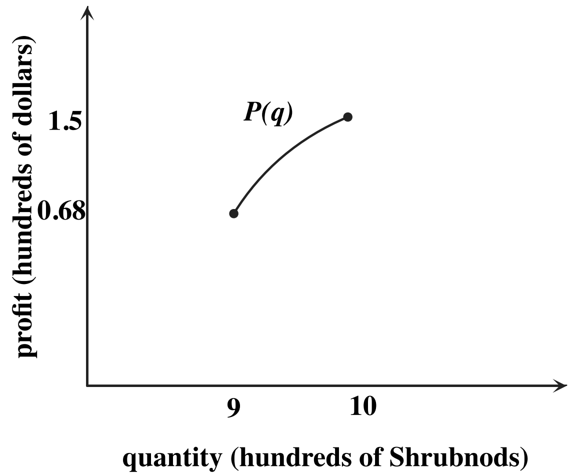
(c) If you produce between 10 hundred and 15 hundred Shrubnods, what production level will yield the largest profit? (Follow the three-step method given above: You've already done Step 1. Evaluate $P(q)$ at any relevant critical values and the endpoints of your interval. Use that information to sketch the graph of $P(q)$ from $q = 10$ to $q = 15$. Then you can see the optima.)
SOLUTION: $P(10) = 1.5$, $P(13) = 2.76$, and $P(15) = 2$



ANSWER: If you produce between 10 hundred and 15 hundred Shrubnods, profit will be maximized if you produce and sell 13 hundred Shrubnods.

(d) If you produce between 9 hundred and 10 hundred Shrubnods, what production level will yield the largest profit?

SOLUTION: $P(9) = 0.68$ and $P(10) = 1.5$

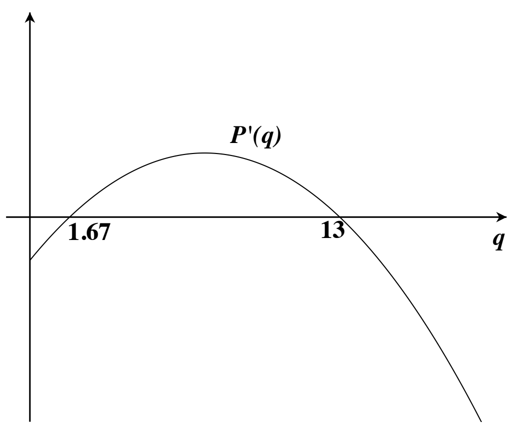


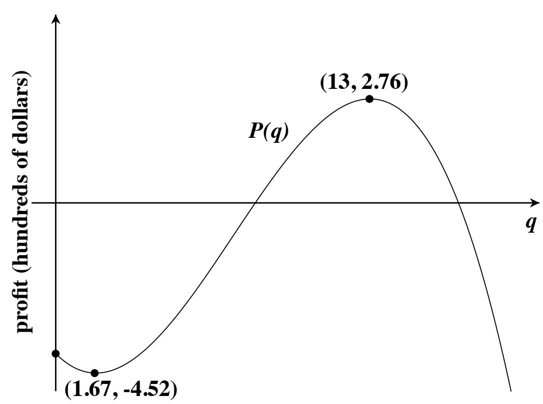
ANSWER: If you produce between 9 hundred and 10 hundred Shrubnods, profit will be maximized if you produce and sell 10 hundred Shrubnods.

(e) If you can produce any number of Shrubnods, what production level will yield the largest profit? What is the largest possible profit for producing Shrubnods? (To answer this question, sketch a rough graph of the entire profit function. Include the correct "y"-intercept and all critical points.)

SOLUTION: To sketch the entire graph of $P(q)$, it helps to look at the graph of $P'(q)$. $P'(q)$ is negative for $0 < q < 1.67$, positive for $1.67 < q < 13$, and negative for $q > 13$. That means that $P(q)$ is decreasing for $0 < q < 1.67$, increasing for $1.67 < q < 13$, and decreasing for $q > 13$.

The "y"-intercept of profit is $P(0) = -4$ and the critical points are $(1.67, -4.62)$ and $(13, 2.76)$.





We can see from the graph of profit that, if we can sell any number of Shrubnods, then the largest possible profit occurs when we sell $q = 13$ hundred of them and the largest possible profit is 2.76 hundred dollars.

(f) What is the global minimum value of profit on the interval from $q = 0$ to $q = 5$ hundred Shrubnods? (NOTE: Your answer should be negative. If the smallest possible profit is negative, then its absolute value is the largest possible loss.)

ANSWER: We can see from the graph in the previous part that the lowest profit on this interval is -4.52 hundred dollars. (So, the largest possible loss on this interval is 4.52 hundred dollars.)

(g) If you can produce any number of Shrubnods, what production level will cause you to lose the most money? What is the largest possible loss for producing Shrubnods?

ANSWER: Again, from the graph of profit, we see that, after $q = 13$, the profit function decreases, getting more and more negative, the larger q gets. There are no other critical values of profit, so profit will never start increasing again. Therefore, there is no greatest possible loss.