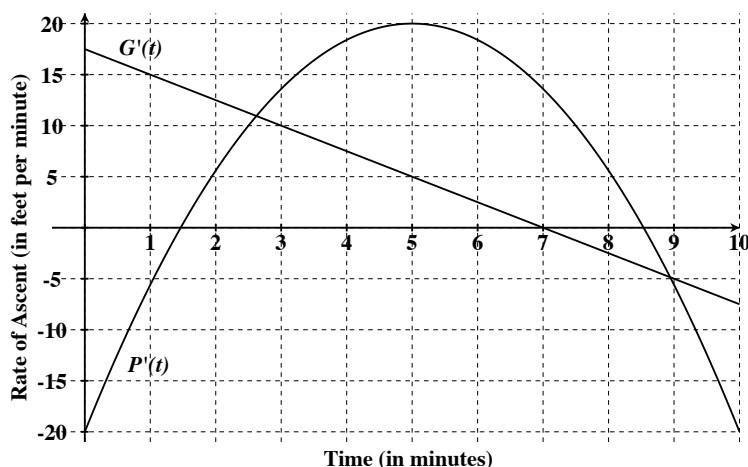


Math 112

Group Activity: Rate of Ascent Graphs

A Purple balloon and a Green balloon rise and fall. When we start watching, at $t = 0$, both balloons are 30 feet above the ground. Their altitudes at time t minutes are given by functions $P(t)$ and $G(t)$, both measured in feet. The graphs below show the *instantaneous rates of ascent* of the balloons. That is, the graphs show $P'(t)$ and $G'(t)$. So, for example, at $t = 1$, the rate of ascent of the Green balloon is 15 feet per minute, which means that the Green balloon's altitude is *increasing* (i.e., the Green balloon is rising) at a rate of 15 feet per minute. Similarly, at $t = 1$, the rate of ascent of the Purple balloon is -5 feet per minute, which means that the Purple balloon's altitude is *decreasing* (i.e., the Purple balloon is falling) at a rate of 5 feet per minute.



- At $t = 0$, the rate graph shows that the Green balloon's rate of ascent is 17.5 and the Purple balloon's rate is -20 . What does that mean about the balloons when we first start observing? Are they rising or falling?

ANSWER: Green is rising at a rate of 17.5 feet per minute and Purple is falling at a rate of 20 feet per minute.

- During the first 1.5 minutes, the Green balloon's rate graph decreases from 17.5 to 13.75. What does that mean about the Green balloon? Is it rising or falling? Is it getting faster or slower?

ANSWER: Green is rising (since its rate is positive) but it's getting slower as it rises.

- The Purple balloon's rate graph is negative during the first 1.5 minutes. What does that mean about the Purple balloon? How does the Purple balloon's rate change during the first 1.5 minutes? Is the balloon getting faster or slower?

ANSWER: Purple is falling during the first 1.5 minutes. Its rate goes from -20 feet per minute to 0 feet per minute. So, it's getting slower and, in fact, it stops for an instant at $t = 1.5$.

- During the first 1.5 minutes are the balloons getting farther apart or closer together? (Think about whether the balloons are rising or falling during this time period.) Which balloon is higher at $t = 1.5$?

ANSWER: At $t = 0$, the balloons have the same altitude. Then Green rises while Purple falls. So, they're getting farther apart during the first 1.5 minutes and Green is higher at $t = 1.5$.

5. You can't tell their exact altitudes by looking at the rate graphs, but suppose that you are told that at $t = 1.5$ minutes, the Green balloon is 54 feet higher than the Purple balloon. Over the next 1.1 minutes, both balloons are rising (how can you tell?) and Green's rate graph is above Purple's rate graph. Are the balloons getting farther apart or closer together from $t = 1.5$ to $t = 2.6$ minutes?

ANSWER: (You can tell that both balloons are rising because their rate graphs are **positive**.) At $t = 1.5$, Green has a higher altitude than Purple and then they both rise but Green's rate is above Purple's. That is, the higher balloon is rising faster than the lower balloon, which means that they're getting farther apart.

6. The two rate graphs cross at 2.6 minutes. This means that the balloons are rising at the same **rate** at that instant. But they do not have the same **altitude** at $t = 2.6$. In fact, since the Green balloon has always been rising faster than the Purple balloon, the two balloons are farther apart at $t = 2.6$ than at any previous time, with the Green balloon much higher than the Purple balloon.

Over the next few minutes, beginning at $t = 2.6$, both balloons are rising but the Purple balloon's rate graph is above the Green balloon's rate graph. Are the balloons getting farther apart or closer together during the next few minutes beginning at $t = 2.6$? Explain.

ANSWER: At $t = 2.6$, the Purple balloon has a lower altitude than Green but, immediately after $t = 2.6$, they're both rising and Purple is rising faster. If both balloons are rising and the lower balloon is rising faster, then they're getting closer together.

7. Do you agree with the following statement?

During the first four minutes, the balloons are farthest apart at $t = 2.6$, when their rate graphs are crossing.

Describe an analogous situation in the context of revenue, cost, and profit.

ANSWER: This is similar to the idea that profit is maximized at a quantity at which the marginal revenue and marginal cost graphs cross. The graphs of total revenue and total cost are like the graphs of altitude and the graphs of marginal revenue (the derivative of TR) and marginal cost (the derivative of TC) are like the graphs of rate of ascent (the derivative of altitude). Profit is the vertical distance between TR and TC and TR and TC are farthest apart when $MR = MC$. The balloons are farthest apart when their rates of ascent are equal.

8. The Green balloon's rate is positive until $t = 7$ and then it becomes negative. Describe what you would see the Green balloon do at $t = 7$.

ANSWER: The Green balloon would stop rising and start falling at $t = 7$.

9. On the interval from $t = 7$ to $t = 8$, determine whether each balloon is rising or falling and whether each balloon is getting faster or slower.

ANSWER: The Purple balloon is rising and getting slower. The Green balloon is falling and getting faster.

10. Again, you can't tell by looking at the rate graphs, but suppose you're told that at $t = 8.5$ minutes, the Purple balloon is approximately 32 feet higher than the Green balloon. Describe what happens to the balloons from $t = 8.5$ to $t = 10$. Are they rising or falling? Getting faster or slower? (BONUS: Do they get closer together or farther apart?)

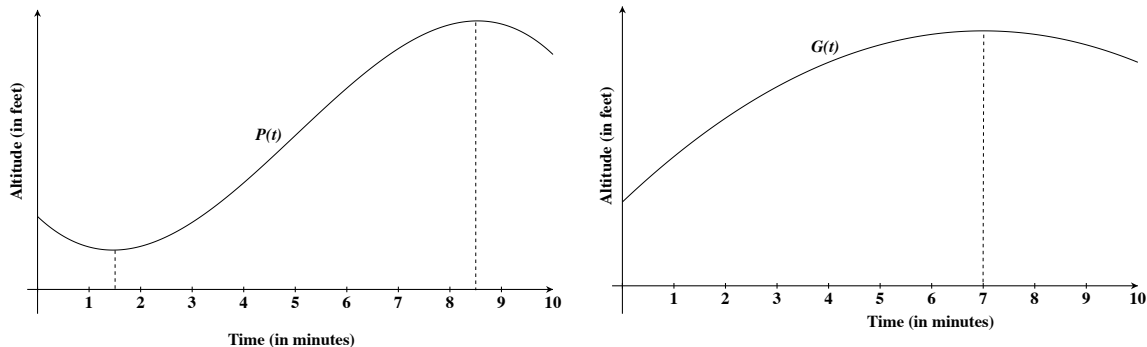
ANSWER: They're both falling and getting faster. At $t = 9$, they have the same speed and that's the instant that Purple starts falling faster than Green.

BONUS: From $t = 8$ to $t = 8.5$, the Green balloon is lower than the Purple balloon and is falling faster. They're getting farther apart. From $t = 8.5$ to $t = 10$, the Green balloon is still lower than the Purple balloon but the Purple balloon is falling faster. So, they're getting closer together.

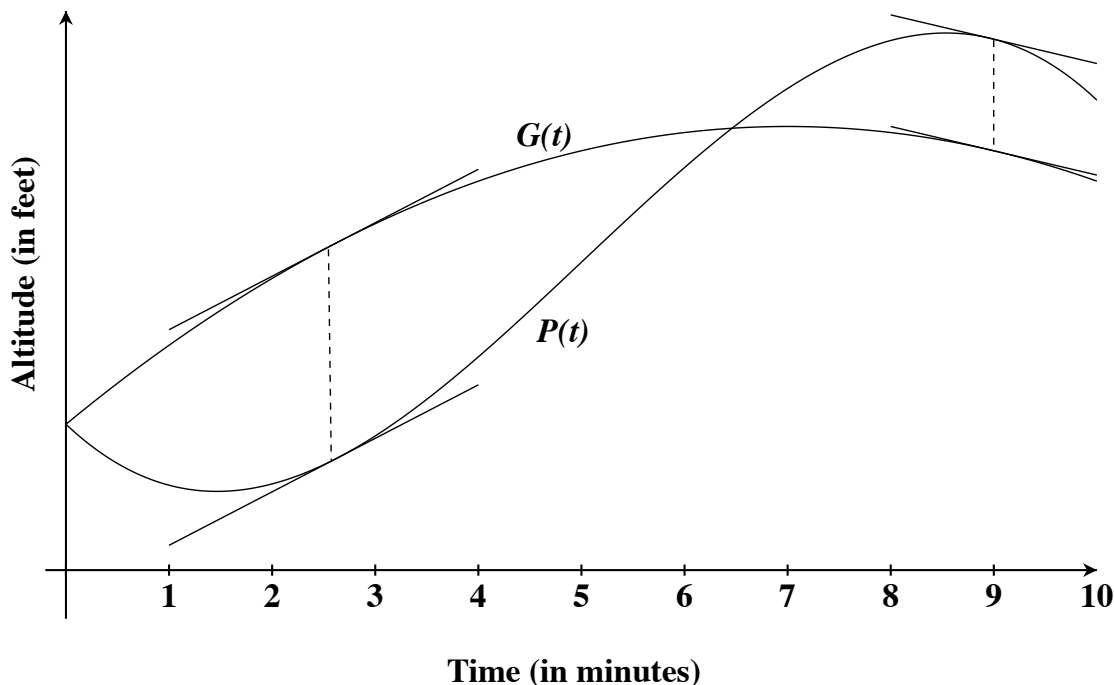
11. On a separate sheet of paper, draw the basic shapes of the altitude graphs for each balloon, each on a separate set of axes. Don't worry about getting the exact altitudes, just note where each graph is increasing and decreasing and the times at which the balloons change direction (from rising to falling or from falling to rising).

ANSWER: The Purple balloon decreases from $t = 0$ to $t = 1.5$, increases from $t = 1.5$ to $t = 8.5$, and decreases from $t = 8.5$ to $t = 10$.

The Green balloon increases from $t = 0$ to $t = 7$ and then decreases from $t = 7$ to $t = 10$.



NOTE: At this point, you don't have the tools that will allow you to draw these graphs on the same set of axes—you can't tell the relative altitudes of the balloons well enough to do that. But here they are:



Notice that the altitude graphs are **farthest apart** at $t = 2.6$ and $t = 9$, the times when the rate graphs **cross**. At these times, the derivatives of the altitude functions are equal. That is, at $t = 2.6$, the tangent line to $G(t)$ is **parallel** to the tangent line to $P(t)$. The tangent lines are also parallel at $t = 9$.