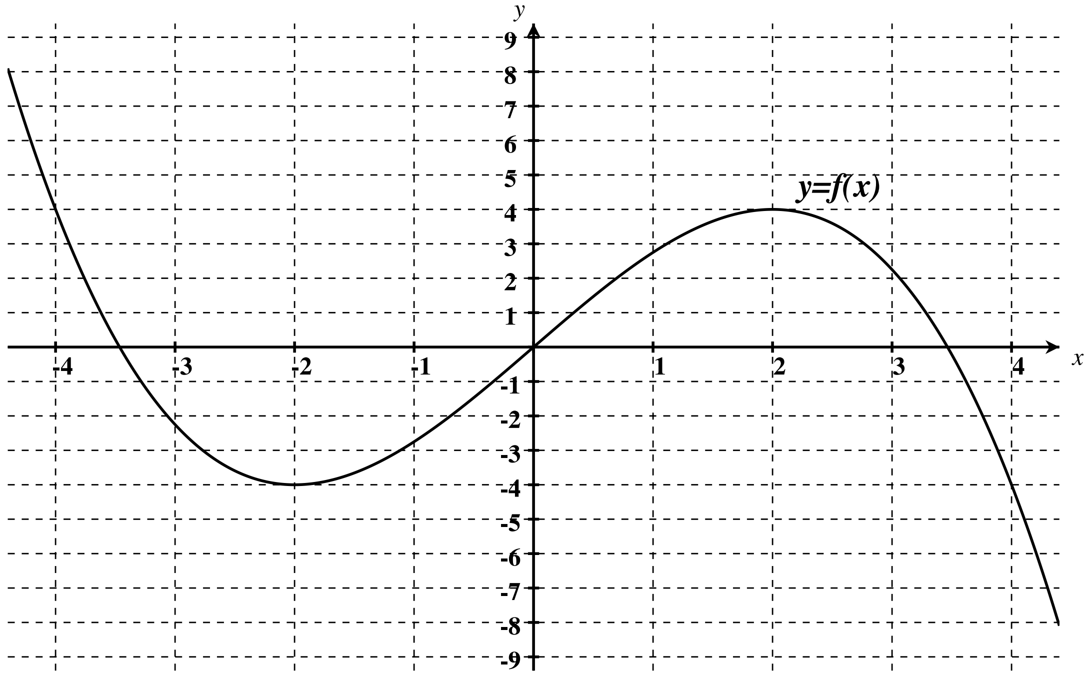
Math 112

Solutions for Group Activity: Graphs of Derivatives

The graph below shows the function $y = f(x)$.



1. Recall that $f'(a)$ gives the slope of the line tangent to $f(x)$ at $x = a$.
   (a) List all values of a at which $f'(a) = 0$.

   ANSWER: $\boxed{a = -2 \text{ and } a = 2}$

   (b) Give four values of a at which $f'(a)$ is *negative*.

   ANSWER: There are many to choose from. Four possibilities are $\boxed{a = -4,\ a = -3,\ a = 3,\ a = 4.}$

   (c) Give three values of a at which $f'(a)$ is *positive*.

   ANSWER: There are many to choose from. Three possibilities are $\boxed{a = -1,\ a = 0,\ a = 1.}$

   (d) Describe how you can tell by looking at the graph of $f(x)$ whether the value of $f'(a)$ will be positive, negative, or zero.

   ANSWER: $f'(a)$ is the slope of the line tangent to $f(x)$ at $x = a$. $f'(a)$ will be positive if $f(x)$ is increasing near $x = a$; $f'(a)$ will be negative if $f(x)$ is decreasing near $x = a$; $f'(a)$ will be 0 if $f(x)$ is "flat" at $x = a$.

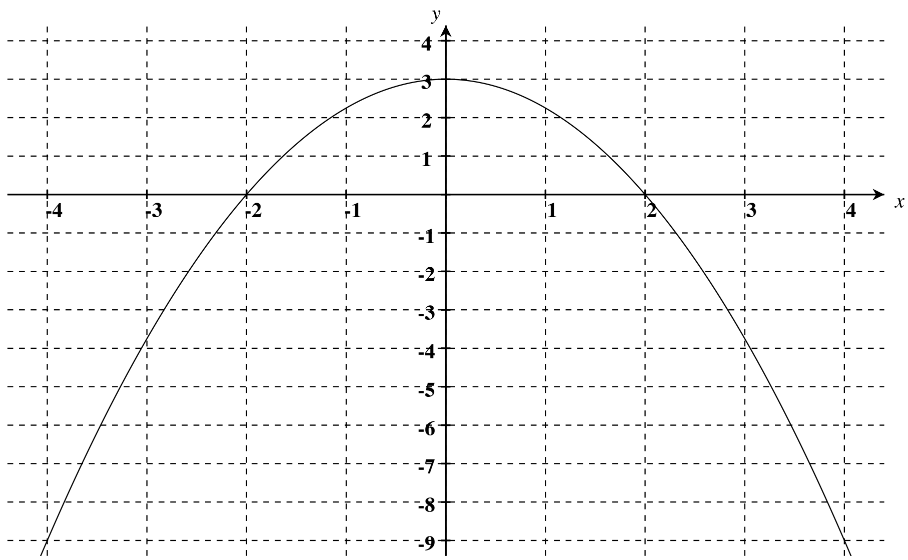
   (e) This graph has some nice symmetry. Without computing any slopes, use your ruler to verify that the following are true for this graph: $f'(-1) = f'(1)$, $f'(-3) = f'(3)$, and $f'(-4) = f'(4)$.

   SOLUTION: Two lines have the same slope when they are *parallel*. Verify that the tangent line to $f(x)$ at $x = -1$ is parallel to the tangent line to $f(x)$ at $x = 1$. Verify that the tangent line to $f(x)$ at $x = -3$ is parallel to the tangent line to $f(x)$ at $x = 3$. Verify that the tangent line to $f(x)$ at $x = -4$ is parallel to the tangent line to $f(x)$ at $x = 4$.

   (f) The following table contains some of the values of $f'(x)$. Fill in the remaining entries in the table. To fill in each entry, either use entries already in the table along with the symmetry described in part (e) or draw an appropriate tangent line and compute its slope.

x	-4	-3	-2	-1	0	1	2	3	4
$f'(x)$	-9	-3.75	0	2.25	3	2.25	0	-3.75	-9

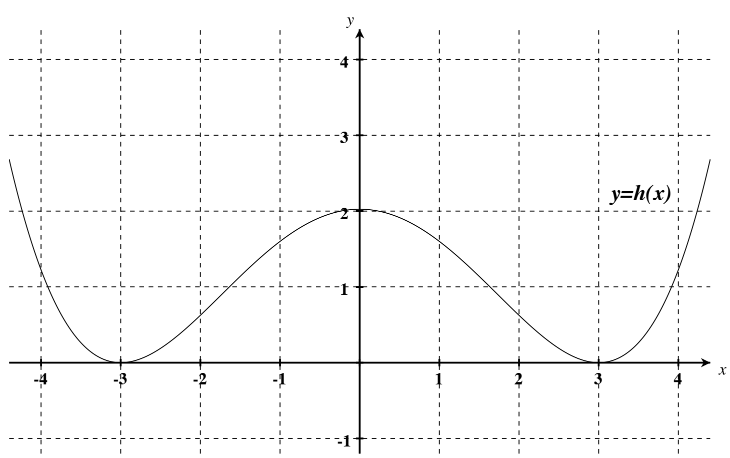
(g) Use the values in the table to sketch the graph of $f'(x)$ on the axes below. Notice that slopes of tangent lines to the graph of $f(x)$ become y-values on the graph of $f'(x)$.



Notice that:

- The graph of $f'(x)$ crosses the x-axis at those values of x where $f(x)$ has horizontal tangent lines.
- The graph of $f'(x)$ is *below* the x-axis (i.e., its y-values are *negative*) on the intervals where the graph of $f(x)$ is *decreasing*.
- The graph of $f'(x)$ is *above* the x-axis (i.e., its y-values are *positive*) on the intervals where the graph of $f(x)$ is *increasing*.

2. Now consider the following graph of a function $h(x)$.



(a) List all values of x at which the graph of $h'(x)$ crosses the x-axis.

ANSWER: $x = -3$, $x = 0$, $x = 3$

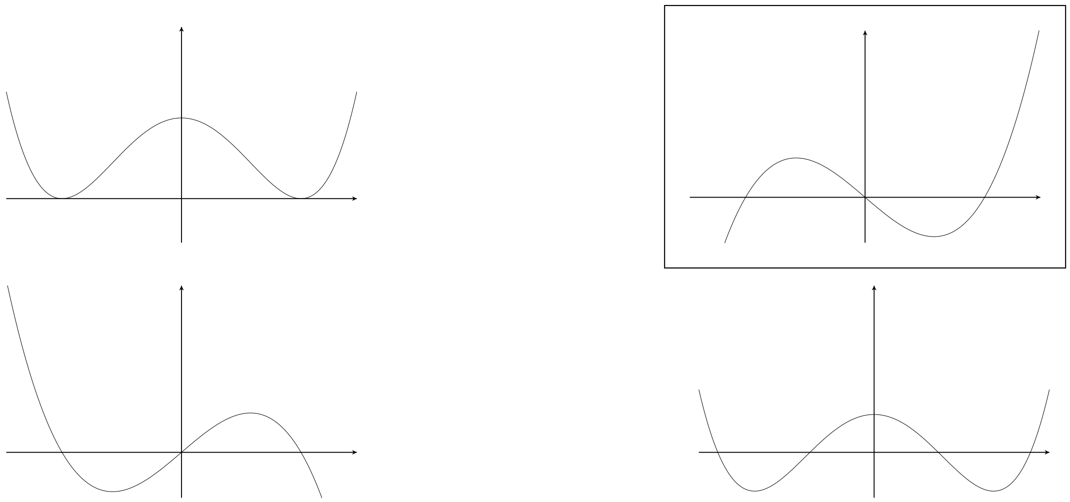
(b) List all intervals on which the graph of $h'(x)$ is *below* the x-axis.

ANSWER: $h'(x)$ is below the x-axis when $h(x)$ is decreasing: for $x < -3$ and from $x = 0$ to $x = 3$.

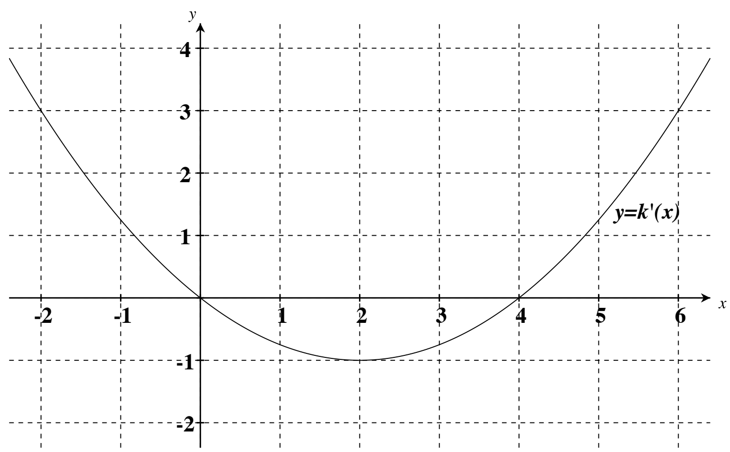
(c) List all intervals on which the graph of $h'(x)$ is *above* the x-axis.

ANSWER: $h'(x)$ is above the x-axis when $h(x)$ is increasing: from $x = -3$ to $x = 0$ and for $x > 3$.

(d) Which of the following is the graph of $h'(x)$?



3. Below is the graph of $k'(x)$, the derivative of a function $k(x)$. The graph of $k(x)$ is not shown.



(a) List all values of x at which the graph of $k(x)$ has horizontal tangent lines.

ANSWER: $k(x)$ has horizontal tangents where $k'(x)$ crosses the x-axis: $\boxed{x = 0 \text{ and } x = 4.}$

(b) List all intervals on which the graph of $k(x)$ is *increasing*.

ANSWER: $k(x)$ is increasing when $k'(x)$ is above the x-axis: $\boxed{\text{for } x < 0 \text{ and } x > 4.}$

(c) List all intervals on which the graph of $k(x)$ is *decreasing*.

ANSWER: $k(x)$ is decreasing when $k'(x)$ is below the x-axis: $\boxed{\text{from } x = 0 \text{ to } x = 4.}$

(d) Which of the following could be the graph of $k(x)$ on the interval from $x = 0$ to $x = 4$?