

From Munkres, *Elements of Algebraic Topology*, p. 407:

1. You may assume this fact: if  $R \rightarrow R'$  is a homomorphism of commutative rings, then for any topological space  $X$ ,  $H^*(X; R) \rightarrow H^*(X; R')$  is a ring homomorphism. Use this plus our knowledge of  $H^*(\mathbf{R}P^n; \mathbf{F}_2)$  to compute the integral cohomology rings of  $\mathbf{R}P^n$  (may depend on the parity of  $n$ ) and  $\mathbf{R}P^\infty$ .
2. Let  $X$  be a connected triangulable homology 7-manifold. Suppose that

$$\begin{aligned} H_7(X) &\cong \mathbf{Z}, & H_6(X) &\cong \mathbf{Z}, \\ H_5(X) &\cong \mathbf{Z}/2, & H_4(X) &\cong \mathbf{Z} \oplus \mathbf{Z}/3. \end{aligned}$$

What can you deduce about the cohomology groups of  $X$  and the cohomology ring of  $X$ ?

3. Let  $X$  be a compact orientable, triangulable homology  $(4n + 2)$ -manifold. Show that  $H_{2n+1}(X) \not\cong \mathbf{Z}$ .