Math 136: Homework 7

Due Thursday, May 24

1. Suppose that $T = f(x, y)$ represents the temperature on the floor of a room at the point (x, y), where x and y are measured in meters (m), and T is measured in degrees Celsius ($°C$). Assume that T is differentiable. You are given the following data:

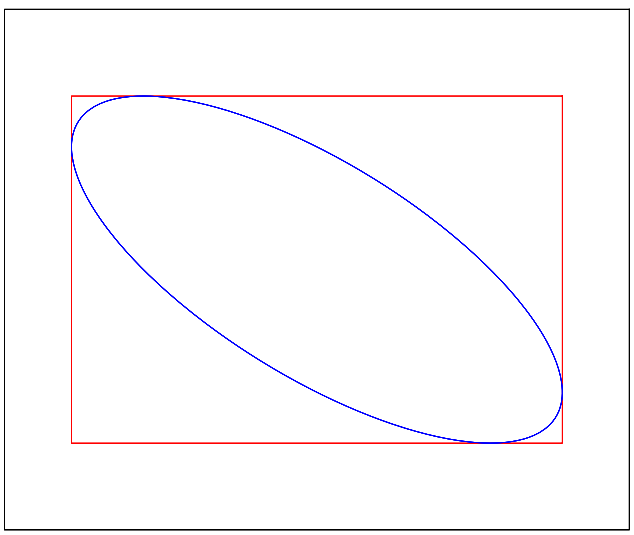
$$f(0,0) = 20°C, \quad f_x(0,0) = 3°C/m \quad f_y(0,0) = 4°C/m\,.$$

You are located at the origin (the center of the floor). Using only the information given, answer the following questions as best you can.

(a) If you are only allowed to move a maximum distance of 1 m (in any direction) along the floor, can you move to a place where the temperature is (approximately) $25°C$? Explain.

(b) Describe the set of points on the floor near you where the temperature is (approximately) $20°C$.

2. Fix a constant $c > 1$ and consider the ellipse $x^2 + 2xy + cy^2 = 1$. Use Lagrange multipliers to find the dimensions of the smallest rectangle, with sides parallel to the axes, containing the ellipse.



3. The *complement* of a set $A \subset \mathbb{R}^n$, written $\mathbb{R}^n \setminus A$ (or sometimes $\mathbb{R}^n - A$ or A^c), is the set

$$\mathbb{R}^n \setminus A = \{\mathbf{x} \in \mathbb{R}^n \,:\, \mathbf{x} \notin A\}.$$

Prove that a set is open if and only if its complement is closed.

4. Let A and B be two open subsets of $\mathbb{R}^n$. Show that their union $A \cup B$ and their intersection $A \cap B$ are also open. Then (using problem 3 and De Morgan's laws[1]) show that if S and T are closed, then $S \cup T$ and $S \cap T$ are also closed.

[1] `http://planetmath.org/DeMorgansLaws.html`