Math 136: Homework 2

Due Thursday, April 5

(1) Exercise 5.3: for any angle θ, write

$$T_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

for the matrix representing rotation of $\mathbf{R}^2$ counterclockwise by angle θ. Fix angles α and β and consider the rotation matrices T_α, T_β, and $T_{\alpha+\beta}$.

(a) Compute the matrix product $T_\alpha T_\beta$.

(b) Explain, geometrically, why $T_\alpha T_\beta = T_{\alpha+\beta}$.

(c) Deduce formulas for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$.

(2) An $n \times n$ matrix $A = (a_{ij})$ is called *upper triangular* if $a_{ij} = 0$ when $i > j$. For example, $\begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & -9 \end{pmatrix}$ is upper triangular. Prove that the product of two $n \times n$ upper triangular matrices is upper triangular.

(3) Let V denote the set of continuous functions on the interval $[0,1]$. It is a vector space under the operations of addition of functions and multiplication by a real number.

(a) Let S be the following subset of V:

$$S = \{f \in V : f(0) = 0\}.$$

Does S form a subspace of V? Justify your answer.

(b) Let T be the following subset of V:

$$T = \{f \in V : f(0) \neq 0\}.$$

Does T form a subspace of V? Justify your answer.

(4) Choose the numbers a, b, c, d, in the following augmented matrix so that (a) there is no solution (b) there are infinitely many solutions to the corresponding system of linear equations:

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & a \\ 0 & 4 & 5 & b \\ 0 & 0 & d & c \end{array}\right)$$