Math 136: Homework 1

Due Thursday, March 29

(1) Exercise 2.5 (p. 11).

(2) Fix a positive integer n.

(a) Recall from Exercise 1.2 on p. 5 that an $n \times n$ matrix A is *symmetric* if $A = A^T$. Show that the collection of symmetric $n \times n$ matrices is a vector space.

(b) An $n \times n$ matrix A is called *antisymmetric* (or *skew-symmetric*) if $A = -A^T$ – see Exercise 2.4. Show that the set of $n \times n$ antisymmetric matrices is a vector space.

(c) Show that the $n \times n$ zero matrix is the only matrix which is both symmetric and antisymmetric.

(3) Fix a positive integer n and let A be an $n \times n$ matrix. The goal of this problem is to show that there exists a unique way to write A as a sum of a symmetric matrix X and an antisymmetric matrix Y.

(a) Show that $A + A^T$ is symmetric and $A - A^T$ is antisymmetric.

(b) Find a symmetric matrix X and an antisymmetric matrix Y so that $A = X + Y$. (This is the first part of the goal: existence.)

(c) Show that if X' and Y' are any matrices with X symmetric and Y antisymmetric, and if $A = X' + Y'$, then $X' = X$ and $Y' = Y$ (with X and Y from part (b)). (This is the second part of the goal: uniqueness.) Hint: if $X + Y = X' + Y'$, then $X - X' = Y' - Y$.