Project: Genetics

General Information. *Genetics* is the study of inheritance. This project examines the specific kind of inheritance known as *autosomal* inheritance: each inherited trait is assumed to be governed by a pair of genes (denoted A and a) and each individual in the population carries one such pair. The possibilities for each individual are AA, Aa, aa, and which of these pairs the individual possesses is called the individual's *genotype*. Genotype determines how the trait manifests in the person. Examples include snapdragon color, eye color, blood type, etc.

Linear algebra provides a way to model how parents pass on the genes to their offspring, and give probabilities for the genotype of offspring given the genotype of the parents. Furthermore, this can be used to follow genotype distribution through many successive generations.

Key Words. Autosomal inheritance, genotype, recessive/dominant gene, eigenvectors and eigenvalues, diagonalization of a matrix.

References. Basic genetics information is covered in most biology books, though you'll probably need to go to a more advanced text to see how linear algebra is involved. For this project, you will also need to learn about eigenvalues and eigenvectors before the rest of the class – check out the book or come to me for help with this.

Problems. In autosomal inheritance, offspring inherit one gene of the pair belonging to each parent in order to get a pair. For the following, assume that either of a parent's genes is equally likely to be inherited.

(1) A farmer has a large population of plants consisting of some distribution of all three possible genotypes: AA, Aa, aa. First show that the following table correctly describes the probabilities of the possible genotypes of the offspring for all possible combinations of the genotypes of the parents.

		Genotypes of Parents					
		AA,AA	*AA,Aa*	*AA,aa*	*Aa,Aa*	*Aa,aa*	*aa,aa*
Genotype	AA	1	.5	0	.25	0	0
of	Aa	0	.5	1	.5	.5	0
Offspring	aa	0	0	0	.25	.5	1

Let a_0, b_0, c_0 denote the portion of the initial population with genotype AA, Aa, aa, respectively.

(2) The farmer undertakes a breeding program in which each plant in the population is always fertilized with a plant of genotype AA.

(a) Derive an expression for the distribution of the three possible genotypes in the population after one generation in terms of the fractions present in the initial population.

(b) Derive an expression for the distribution of the three possible genotypes in the population after n generations in terms of the fractions present after $n-1$ generations.

(c) Use the above information and eigenvectors and eigenvalues to find an expression for the distribution of the three possible genotypes in the population after n generations in terms of the fractions present in the initial population.

(3) Another farmer modifies the fertilization procedure by undertaking a breeding program in which each plant in the population is fertilized with a plant of its own genotype.

(a) Use the steps outlined in problem 2 to derive an expression for the distribution of the three possible genotypes in the population after n generations.

(b) Find the fractions of the population of each genotype after 5 generations.

(c) Find the limiting genotype distribution as n goes to infinity.