PROJECT: GAME THEORY

General Information. *Game theory* is used to model 2-person (or 2-player) games requiring the same decisions to be made at each step. Furthermore, these decisions may result in payoffs or penalties for each player at each step. In such situations, it is often possible for each player to formulate a strategy which will maximize their return. Linear algebra can be used to describe this situation. In this project, you will learn exactly how linear algebra enters into formulating strategies.

Key Words. Two person zero-sum game, payoff matrix, strategy, expected payoff, optimal strategy, value of a game, saddle point, fundamental theorem of a 2-person zero-sum game

References. Basic books on operations research (a branch of applied mathematics which studies these types of problems) are a good place to look for information.

Problems.

(1) Suppose that a game pays off according to the following table:

		B			
		i	ii	iii	iv
	i	-4	6	-4	1
A	ii	5	-7	3	8
	iii	-8	0	6	-2

   (a) Suppose that player A uses strategy i half of the time, strategy iii half of the time, and strategy ii none of the time. Suppose also that player B uses each of the four strategies one fourth of the time. Find the expected payoff of the game.

   (b) If player B keeps his strategy the same as in part (a), what strategy should player A choose to maximize her expected payoff?

   (c) If player A keeps her strategy the same as in part (a), what strategy should player B choose to maximize his expected payoff?

(2) Two clothing stores in a shopping center compete for the weekend trade. On a clear day the larger store gets 60% of the business and on a rainy day the larger store gets 80% of the business. Either or both stores may hold a sidewalk sale on a

given weekend, but the decision must be made a week in advance and in ignorance of the competitor's plans. If both have a sidewalk sale, each gets 50% of the business. If, however, one holds the sale and the other doesn't, the one conducting the sale gets 90% of the business on a clear day and 10% on a rainy day. It rains 40% of the time. How frequently should each retailer conduct sales?