Math 135: Homework 9

Due Thursday, March 8

(1) Let ℓ_1 and ℓ_2 be two skew (i.e. not parallel) line with parameterizations

$$\ell_j : \quad \mathbf{r}(t) = \mathbf{r}_j + t\,\mathbf{d}_j, \quad j = 1, 2$$

Show that the distance between the lines is given by the formula

$$\mathrm{d}(\ell_1, \ell_2) = \frac{|(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{d}_1 \times \mathbf{d}_2)|}{\|\mathbf{d}_1 \times \mathbf{d}_2\|}$$

(2) A vector-valued function $\mathbf{G}$ is called an *antiderivative* for $\mathbf{f}$ on $[a, b]$ provided that $\mathbf{G}$ is continuous on $[a, b]$ and $\mathbf{G}'(t) = \mathbf{f}(t)$ for all $t \in (a, b)$.

(a) Show that if $\mathbf{f}$ is continuous on $[a, b]$ and $\mathbf{G}$ is an antiderivative for $\mathbf{f}$ on $[a, b]$ then

$$\int_a^b \mathbf{f}(t)\,dt = \mathbf{G}(b) - \mathbf{G}(a).$$

(b) Show that if $\mathbf{f}$ is continous on $[a, b]$ and $\mathbf{F}$ and $\mathbf{G}$ are antiderivatives for $\mathbf{f}$ on $[a, b]$ then

$$\mathbf{F} = \mathbf{G} + \mathbf{C}$$

for some constant vector $\mathbf{C}$.

(3) Three objects move in space according to the equations

$$\mathbf{r} = \mathbf{r}_1(t), \quad \mathbf{r} = \mathbf{r}_2(t), \quad \text{and } \mathbf{r} = \mathbf{r}_3(t),$$

where t denotes time. Let $A(t)$ denote the area of the triangle formed by the three objects. Suppose that

$$\begin{array}{lll} \mathbf{r}_1(0) = \mathbf{i} + \mathbf{j} + \mathbf{k} & \mathbf{r}_2(0) = \mathbf{i} + \mathbf{j} - \mathbf{k} & \mathbf{r}_3(0) = \mathbf{k} \\ \mathbf{r}_1'(0) = \mathbf{i} & \mathbf{r}_2'(0) = \mathbf{j} & \mathbf{r}_3'(0) = \mathbf{k} \end{array}$$

Compute $A'(0)$.