Math 135: Homework 4

Due Thursday, February 2

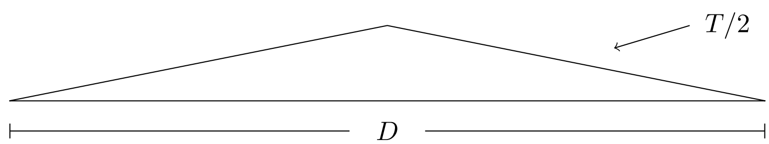
(1) Problem 66 on page 613 of the text.
Note: *There is a typo:* s_q should be defined by $s_q = \sum_{k=0}^{q} \frac{1}{k!}$.

(2) A widget company has a store exactly one mile (5280 feet) from its factory, and they want to move their widgets from factory to store by train. So they order a one-mile long piece of railroad track, but mistakes are made, and they get a piece of track that is too long by one foot: it is 5281 feet long. They try to jam it in place anyway, hammering the ends down and letting it bulge up in the middle. How far off the ground will it be in the center? (Ignore the curvature of the earth.)

This is an open-ended question, so I'll describe two cases. In all three, let T denote the length of the track (so $T = 5281$) and let D denote the distance from the factory to the store (so $D = 5280$).

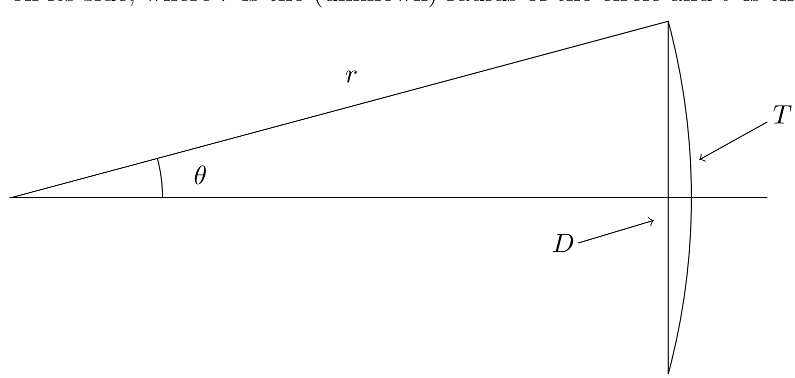
Before proceeding, you might want to take a guess at the answer. Will it be about an inch? About a foot? About a mile?

(a) Assume that the train track forms two straight line segments, making a triangle:



Calculate the height of this triangle.

(b) Assume that the train track forms the arc of a circle. Here's a picture turned on its side, where r is the (unknown) radius of the circle and θ is the marked angle:



In terms of r and θ, what quantity are you trying to find?

Write down equations relating r, θ, D, and T. If you could solve for the unknowns θ and r in terms of the knowns D and T, you could find the height of the top of the track. You can easily find $r\theta$, so you really just need to solve for θ. Solving algebraically seems to be impossible, though, so make an *approximation*: θ is close to zero (justify this!), so you can replace $\sin\theta$ or $\cos\theta$ with the first few terms of their Maclaurin series. Now solve for θ.