(1) Let $U \subset \mathbb{R}^4$ be the subspace spanned by the two column vectors

$$A_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \; A_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}.$$

Let $P : \mathbb{R}^4 \to \mathbb{R}^4$ denote the linear map given by orthogonal projection onto U. Find the matrix of P with respect to the standard basis for $\mathbb{R}^4$.

(2) Problem #5 on page 208 of Lang.

(3) Let $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$, and $P_3 = (x_3, y_3)$ be three non-collinear points in $\mathbb{R}^2$. Show that the equation of the circle passing through these points is given by the equation

$$\begin{vmatrix} 1 & x & y & x^2 + y^2 \\ 1 & x_1 & y_1 & x_1^2 + y_1^2 \\ 1 & x_2 & y_2 & x_2^2 + y_2^2 \\ 1 & x_3 & y_3 & x_3^2 + y_3^2 \end{vmatrix} = 0.$$

(4) Let n be odd and let A be an $n \times n$ skew-symmetric matrix.

(a) Prove that $\det(A) = 0$.

(b) Prove that the map $L_A : \mathbb{R}^n \to \mathbb{R}^n$ is not surjective.