Math 136: Homework 3

Due Thursday, April 14

(1) Let V be the vector space of infinitely differentiable functions on the unit interval. Show that V is not finite dimensional. [**Hint:** Consider the functions $f_k(x) = x^k$, $k = 0, 1, 2, 3, \ldots$.]

(2) Let V be the vector space of infinitely differentiable functions on the real line and fix $p, q \in V$.

   (a) Show that the map $L : V \to V$ defined by

$$L(f) = f'' + pf' + qf$$

   is a linear map.

   (b) What can you say about the kernel of L?

(3) Let $M(n)$ be the vector space of $n \times n$ matrices. The *Lie bracket* of two matrices A and B in $M(n)$ is the matrix

$$[A, B] = AB - BA$$

   (So $[A, B] = 0 \iff AB = BA$.)

   Choose a matrix A, and consider the map

$$\mathcal{L}_A : B \mapsto [A, B]$$

   Show that $\mathcal{L}_A$ is a linear map. Describe the kernel of $\mathcal{L}_A$.

(4) Let V be the vector space of infinitely differentiable functions on the interval $[0, 1]$. Show that the map $T : V \to V$ defined by

$$(Tf)(x) = \int_0^x f(t)\, dt$$

   is a linear map. Prove that T is injective. Is T surjective? Explain your answer.