Math 136: Homework 2

Due Thursday, April 7

(1) Choose the numbers a, b, c, d, in the following augmented matrix so that (a) there is no solution (b) there are infinitely many solutions to the corresponding system of linear equations:

$$A' = \begin{pmatrix} 1 & 2 & 3 & a \\ 0 & 4 & 5 & b \\ 0 & 0 & d & c \end{pmatrix}$$

(2) Prove that the product of two $n \times n$ upper triangular matrices is upper triangular.

(3) Let V denote the set of continuous functions on the interval $[0, 1]$. It is a vector space under the operations of addition of functions and multiplication by a real number – see Lang, Example 6 on p. 92.

(a) Let S be the following subset of V:

$$S = \{f \in V : f(0) = 0\}.$$

Does S form a subspace of V? Justify your answer.

(b) Let T be the following subset of V:

$$T = \{f \in V : f(0) \neq 0\}.$$

Does T form a subspace of V? Justify your answer.

(4) Let $M(n)$ be the set of $n \times n$ matrices. Lang points out in Example 1 on p. 89 that $M(n)$ is a vector space.

(a) Recall from p. 46 that a square matrix A is *symmetric* if $A = {}^{t}A$. Show that the subset $S(n) \subset M(n)$ of symmetric matrices is a subspace.

(b) Recall from p. 47 that a square matrix A is *skew-symmetric* if $A = -{}^{t}A$. Show that the subset $A(n) \subset M(n)$ of skew-symmetric matrices is a subspace.

(c) Show that $M(n) = S(n) + A(n)$ and $S(n) \cap A(n) = \{O\}$.