Math 136: Homework 1

Due Thursday, March 31

(1) Let A_1, A_2, ..., A_r be non-zero vectors in $\mathbf{R}^n$ which are mutually perpendicular (i.e. $A_i \cdot A_j = 0$ for $i \neq j$). Let c_1, c_2, ..., c_r be numbers such that

$$c_1 A_1 + c_2 A_2 + \cdots + c_r A_r = O\,.$$

Show that $c_i = 0$ for all i.

(2) Let X and Y be vectors in $\mathbf{R}^n$, $Y \neq O$. Prove that $\|X + Y\| = \|X\| + \|Y\|$ if and only if $X = \lambda Y$ for $\lambda \geq 0$.

(3) Let X and Y be vectors in $\mathbf{R}^n$. Prove the identity

$$\|X + Y\|^2 - \|X - Y\|^2 = 4X \cdot Y\,.$$