PROJECT: POPULATION GROWTH

General Information. A common model used by demographers for studying population growth is the *Leslie model*, which was developed in the 1940s. In this model, the females are divided into age classes of equal duration. Over time, the number of females in each age class changes due to birth, death, and aging. This project will use linear algebra to project what future age distributions will be.

Key Words. Leslie model of population growth, eigenvalues and eigenvectors, diagonalization of matrices, positive and negative eigenvalues, net reproduction rate.

References. Try books on demography or population growth models. For this project, you will also need to learn about eigenvalues and eigenvectors before the rest of the class – check out the book or come to me for help with this.

Problems.

(1) Suppose a certain animal population is divided into two age classes and has a Leslie matrix

$$L = \begin{bmatrix} 1 & 1.5 \\ .5 & 0 \end{bmatrix}.$$

  (a) Calculate the positive eigenvalue of L and the corresponding eigenvector.

  (b) Beginning with the initial age distribution vector

$$\mathbf{x} = \begin{bmatrix} 100 \\ 0 \end{bmatrix},$$

    calculate the age distribution after 1, 2, 3, 4, and 5 years.

  (c) Calculate the age distribution after 6 years in two ways: once by using L, and once by using the eigenvalue you found in (a).

(2) Show that the net reproduction rate can be interpreted as the average number of daughters born to a single female during her expected lifetime. Using this interpretation, reason that a population is eventually decreasing if and only if its net reproduction rate is less than one.

(3) Suppose the oldest age attained by the females of a certain breed of dog is 15 years. Divide the population into three age classes of length 5 years: 0-5 years, 5-10 years, 10-15 years. Suppose that dogs under the age of 5 have 3 puppies on average per year, dogs between 5 and 10 years have 4 puppies on average per year, and dogs between 10 and 15 years have no puppies. Further, suppose that nine-tenths of dogs under the age of 5 are expected to live to the next year, while only three-fourths of dogs between the ages of 5 and 10 years are expected to live to the next year.

(a) Find the Leslie matrix for this population.

(b) If there are originally 1000 females in each age class, find the population after 2 years.

(c) Find the net reproductive rate.