Midterm 2 review

Topics:

- separable equations, autonomous equations
- linear first order equations and their integrating factors
- Picard iteration
- linear degree n equations, linear independence, form of the general solution in both the homogeneous and non-homogeneous cases
- solving linear constant coefficient homogeneous equations using the characteristic equation
- finding a second solution to a linear homogeneous equation using reduction of order
- solving linear constant coefficient nonhomogeneous equations using the method of undetermined coefficients and using variation of parameters
- Laplace transforms and inverse Laplace transforms: basic formulas, using tables, discontinuous forcing functions, the Dirac δ-function (NOT COVERED in 2011: convolutions)
- solving linear constant coefficient equations using Laplace transforms
- (NOT COVERED in 2011) power series solutions, method of Frobenius

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Two old Math 135 midterms

(A1) Find the general solution of the differential equation $y'' + y = 2t^2 + t$.

(A2) Find a particular solution of the differential equation $y'' - y = (1 + t^2)^{-1}$. Express your answer in terms of integrals, and do not attempt to evaluate these integrals.

(A3) Is there a differential equation of the form $y'' + p(t)y' + q(t)y = 0$, with p and q continuous on the open interval $-1 < t < 1$, for which the functions $y = t^4$ and $y = t^6$ are both solutions? Explain your answer.

(A4) The solution of the initial value problem

$$y'' + (t + 1)y' + y = 0\,, \quad y(0) = 1 \quad y'(0) = 0$$

can be expressed in the form $y = \sum_{k=0}^{\infty} a_k t^k$. Find the recursion formula for the coefficients.

(A5) (a) Find $\mathcal{L}^{-1}\left[\frac{1}{(s-1)^2}\right]$.

(b) Use the result of part (a) to find $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{(s-1)^2}\right]$.

(c) Use the results of parts (a) and (b) to solve the initial value problem

$$y'' - 2y' + y = \delta(t - 3)\,, \quad y(0) = 0\,, \quad y'(0) = 1\,.$$

(A6) By computing Laplace transforms, show that the solution of the initial value problem

$$y'' - y = f(t)\,, \quad y(0) = y'(0) = 0$$

can be written as a convolution of f with another function. What is that other function?

(B1) Let c be a constant, and let $y(t)$ be the solution to the initial value problem

$$y'' - 4y = \delta(t-1) + c\delta(t-3), \quad y(0) = 0,\ y'(0) = 0.$$

For what value(s) of c does $y(t)$ satisfy $\lim_{t\to\infty} y(t) = 0$?

(B2) The solution to the initial value problem

$$(1+x^2)y'' + y' - 2y = 0, \quad y(0) = 1,\ y'(0) = 2,$$

can be expressed in the form $y = \sum_{n=0}^{\infty} a_n x^n$. Find the recursion formula for the coefficients. Also find a_0, a_1, a_2 and a_3.

(B3) Find the general solution to the equation

$$y'' + 2y' + 2y = 2\cos t.$$

(B4) Consider the equation

$$y'' - 4y' + 4y = f(t).$$

(a) What is y_c, the general solution to the associated homogeneous equation?

(b) Let $f(t) = (t^2 - 7)\cos 2t$. According to the method of undetermined coefficients, what should you try for y_p? (Don't solve for the coefficients: just give me the form for y_p.)

(c) Let $f(t) = te^{2t}$. According to the method of undetermined coefficients, what should you try for y_p? (Don't solve for the coefficients: just give me the form for y_p.)

(B5) Let $f(t)$ be a continuous function, and suppose that $y_1(t)$ and $y_2(t)$ are solutions to the equation

$$y'' + 4y = f(t)$$

satisfying the initial conditions

$$y_1(0) = 2,\ y_1'(0) = 2, \quad y_2(0) = 2,\ y_2'(0) = 0.$$

Find $y_1(t) - y_2(t)$.

(B6) Find the general solution to the equation

$$x^2y' + xy = 3x^3.$$