

Math 135: Homework 9

Due Thursday, March 3

- (1) Show that for any vectors  $\mathbf{a}$  and  $\mathbf{b}$ ,

$$| \|\mathbf{a}\| - \|\mathbf{b}\| | \leq \|\mathbf{a} - \mathbf{b}\|$$

(Hint:  $\mathbf{a} = (\mathbf{a} - \mathbf{b}) + \mathbf{b}$ .)

- (2) Let  $T$  denote the tetrahedron centered at the origin  $O$  with vertices at the points  $P_1(1, 1, 1)$ ,  $P_2(-1, -1, 1)$ ,  $P_3(1, -1, -1)$ , and  $P_4(-1, 1, -1)$ . Using vector methods, find the cosines of the angles  $\angle P_i O P_j$  for all  $i \neq j$ . What are the approximate angles (in degrees)?

- (3)

(a) Suppose that  $\mathbf{a} \cdot \mathbf{b} = 0$  and  $\mathbf{a} \times \mathbf{b} = \mathbf{0}$ . Show that either  $\mathbf{a} = \mathbf{0}$  or  $\mathbf{b} = \mathbf{0}$ .

(b) Now suppose that  $\mathbf{a} \neq \mathbf{0}$ . If  $\mathbf{c}$  and  $\mathbf{d}$  are vectors for which

$$\mathbf{a} \cdot \mathbf{c} = \mathbf{a} \cdot \mathbf{d} \text{ and } \mathbf{a} \times \mathbf{c} = \mathbf{a} \times \mathbf{d},$$

use the result of (a) to prove that  $\mathbf{c} = \mathbf{d}$ .