

Math 135: Homework 5

Due Thursday, February 3

1. **Coffee problem.** A man and a woman go to a coffee shop and buy identical cups of coffee which they receive at the same time. The man adds some cream to his coffee right away, but does not drink the coffee yet. The woman waits 10 minutes and then adds some cream (the same amount, at the same temperature) to hers. Then they both drink. Whose coffee is hotter?
2. **A snowplow problem.** It starts to snow at noon, and snows at a constant rate the rest of the day. At 1:00, a snowplow leaves the garage and starts plowing. At 2:00, a second plow leaves the same garage and starts plowing along the same route. Does the second plow ever catch up to the first? Suppose the second plow left at 3:00 instead; would that make a difference? (You may assume that the speed of each plow is inversely proportional to the depth of the snow right in front of the plow.)
[Hint: treat time as the dependent variable, distance as the independent variable.]
3. Fix a real constant λ and consider the second order differential equation

$$y'' + \lambda y = 0.$$

Let a and b be real numbers.

- (a) Check that $y = ax + b$ satisfies the equation when $\lambda = 0$.
- (b) Check that $y = a \cos(\sqrt{\lambda}x) + b \sin(\sqrt{\lambda}x)$ satisfies the equation when $\lambda > 0$.
- (c) Check that $y = a \cosh(\sqrt{-\lambda}x) + b \sinh(\sqrt{-\lambda}x)$ satisfies the equation when $\lambda < 0$.