Math 135: Homework 10

Due Thursday, March 10

(1) A vector-valued function $\mathbf{G}$ is called an *antiderivative* for $\mathbf{f}$ on $[a, b]$ provide that $\mathbf{G}$ is continuous on $[a, b]$ and $\mathbf{G}'(t) = \mathbf{f}(t)$ for all $t \in (a, b)$.

(a) Show that if $\mathbf{f}$ is continuous on $[a, b]$ and $\mathbf{G}$ is an antiderivative for $\mathbf{f}$ on $[a, b]$ then
$$\int_a^b \mathbf{f}(t)\, dt = \mathbf{G}(b) - \mathbf{G}(a)$$

(b) Show that if $\mathbf{f}$ is continous on $[a, b]$ and $\mathbf{F}$ and $\mathbf{G}$ are antiderivatives for $\mathbf{f}$ on $[a, b]$ then
$$\mathbf{F} = \mathbf{G} + \mathbf{C}$$
for some constant vector $\mathbf{C}$.

(2) Three objects move in space according to the equations
$$\mathbf{r} = \mathbf{r}_1(t) \quad \mathbf{r} = \mathbf{r}_2(t) \quad \text{and } \mathbf{r} = \mathbf{r}_3(t)\,,$$
where t denotes time. Let $A(t)$ denote the area of the triangle formed by the three objects. Suppose that
$$\begin{array}{lll} \mathbf{r}_1(0) = \mathbf{i} + \mathbf{j} + \mathbf{k} & \mathbf{r}_2(0) = \mathbf{i} + \mathbf{j} - \mathbf{k} & \mathbf{r}_3(0) = \mathbf{k} \\ \mathbf{r}_1'(0) = \mathbf{i} & \mathbf{r}_2'(0) = \mathbf{j} & \mathbf{r}_3'(0) = \mathbf{k} \end{array}$$
Compute $A'(0)$.