Math 135: Homework 1

Due Thursday, January 6

1. Let $\{a_n\}$ be a bounded sequence. Prove that if $\{a_n\}$ is nonincreasing then it converges to its greatest lower bound.

2. Let $\{a_n\}$ and $\{b_n\}$ be sequences such that $a_n \to 0$ and $\{b_n\}$ is bounded. Prove that $a_n b_n \to 0$.

3. Suppose that f is a differentiable function on $(0, \infty)$ such that $f'(x) \to 0$ as $x \to \infty$. Show that

$$\lim_{n \to \infty} (f(n+1) - f(n)) = 0\,.$$

For instance, $\sqrt{n+1} - \sqrt{n} \to 0$ as $n \to \infty$ even though $\sqrt{n} \to \infty$ as $n \to \infty$.

Hint: *Mean value theorem.*