Math 134: Homework 9

Due December 2

(1) By the Fundamental Theorem of Calculus, if f has a continuous first derivative on the interval $[a, b]$ then

$$f(b) = f(a) + \int_a^b f'(x)\,dx\,.$$

(a) Assume that f has a continuous second derivative on $[a, b]$, and apply integration by parts to the integral above to derive the identity

$$f(b) = f(a) + f'(a)(b-a) - \int_a^b f''(x)(x-b)\,dx\,.$$

(b) Finally assume that f has a continuous third derivative on $[a, b]$ and apply integration by parts once more to derive the identity

$$f(b) = f(a) + f'(a)(b-a) + \frac{f''(a)}{2}(b-a)^2 + \frac{1}{2}\int_a^b f'''(x)(x-b)^2\,dx\,.$$

(2) Evaluate the integral $\displaystyle\int \frac{x^2}{(x^2+4x+5)^{3/2}}\,dx$.

(3) Evaluate the integral $\displaystyle\int \frac{4\arctan x}{(x+1)^3}\,dx$.