Math 134: Homework 3

Due October 14

1. Suppose that f is continuous on $[0, 1]$ and takes values in $[0, 1]$; that is, for all $x \in [0, 1]$, we have $0 \leq f(x) \leq 1$. Prove that there is a $c \in [0, 1]$ such that $f(c) = c$. Such a point is called a *fixed point* of f. (Hint: Draw a picture. Consider $f(x) - x$.)

2. Let n be a positive integer.

   (a) Prove that for real numbers a and b, if $0 \leq a < b$, then $a^n < b^n$. (Hint: Use mathematical induction.)

   (b) Prove that for every nonnegative real number x, there is a unique nonnegative n^{th} root, $x^{1/n}$. (Hint: The existence follows from the intermediate value theorem. Use part (a) to get uniqueness.)