

Mathematics 135 Quiz 8

Name: _____ Answers _____

March 11, 2010

Instructions: This is a closed book quiz, no notes or calculators allowed. Please turn off all cell phones, pagers, etc.

1. Consider the curve defined parametrically by

$$\mathbf{r}(t) = e^{-t} \cos t \mathbf{i} + e^{-t} \sin t \mathbf{j} + e^{-t} \mathbf{k}.$$

At each time t , find its unit tangent vector \mathbf{T} and its curvature κ .

Solution: Since the curve is given by

$$\mathbf{r}(t) = e^{-t} \cos t \mathbf{i} + e^{-t} \sin t \mathbf{j} + e^{-t} \mathbf{k},$$

its derivative is

$$\mathbf{r}'(t) = (-e^{-t} \cos t - e^{-t} \sin t) \mathbf{i} + (-e^{-t} \sin t + e^{-t} \cos t) \mathbf{j} - e^{-t} \mathbf{k}.$$

The length of this is

$$\begin{aligned} \|\mathbf{r}'(t)\| &= \left((-e^{-t} \cos t - e^{-t} \sin t)^2 + (-e^{-t} \sin t + e^{-t} \cos t)^2 + (-e^{-t})^2 \right)^{1/2} \\ &= (2e^{-2t} \cos^2 t + 2e^{-2t} \sin^2 t + e^{-2t})^{1/2} \\ &= \sqrt{3}e^{-t}. \end{aligned}$$

Therefore the unit tangent vector is

$$\mathbf{T} = \frac{1}{\sqrt{3}} (-\cos t - \sin t) \mathbf{i} + \frac{1}{\sqrt{3}} (-\sin t + \cos t) \mathbf{j} - \frac{1}{\sqrt{3}} \mathbf{k}.$$

The curvature is

$$\begin{aligned} \kappa &= \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} \\ &= \frac{1}{\sqrt{3}e^{-t}} \|\mathbf{T}'(t)\| \\ &= \frac{e^t}{\sqrt{3}} \left\| \frac{1}{\sqrt{3}} (\sin t - \cos t) \mathbf{i} + (-\cos t - \sin t) \mathbf{j} \right\| \\ &= \frac{e^t}{3} \sqrt{\sin^2 t + \cos^2 t + \sin^2 t + \cos^2 t} \\ &= \frac{\sqrt{2}e^t}{3} \end{aligned}$$