Mathematics 135 Quiz 7 Name: Answers

March 4, 2010

Instructions: This is a closed book quiz, no notes or calculators allowed. Please turn off all cell phones, pagers, etc.

1. Simplify the expression

$$(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b}).$$

Explain your steps.

Solution:

$$\begin{aligned}(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b}) &= \mathbf{a}\times(\mathbf{a}-\mathbf{b})+\mathbf{b}\times(\mathbf{a}-\mathbf{b}) \quad \text{(distributivity)}\\ &= \mathbf{a}\times\mathbf{a}-\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{a}-\mathbf{b}\times\mathbf{b} \quad \text{(distributivity)}\\ &= \mathbf{0}-\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{a}-\mathbf{0} \quad (\text{since } \mathbf{a}\times\mathbf{a}=\mathbf{0})\\ &= -\mathbf{a}\times\mathbf{b}-\mathbf{a}\times\mathbf{b} \quad (\text{since } \mathbf{b}\times\mathbf{a}=-\mathbf{a}\times\mathbf{b})\\ &= -2(\mathbf{a}\times\mathbf{b}).\end{aligned}$$

Of course, $2(\mathbf{b}\times\mathbf{a})$ is an equally valid answer.

2. Suppose that $d_1D_1+d_2D_2+d_3D_3=0$. What can you conclude about the lines

$$\frac{x-x_0}{d_1}=\frac{y-y_0}{d_2}=\frac{z-z_0}{d_3}, \quad \frac{x-x_0}{D_1}=\frac{y-y_0}{D_2}=\frac{z-z_0}{D_3}?$$

Solution: First, the lines intersect: they both go through the point (x_0, y_0, z_0). The equation $d_1D_1+d_2D_2+d_3D_3=0$ says that the dot product of their direction vectors (d_1, d_2, d_3) and (D_1, D_2, D_3) is zero. Since the dot product being zero means that the vectors are perpendicular, we conclude that the lines intersect at right angles to each other.