Mathematics 135 Quiz 6 Name: Answers

February 18, 2010

Instructions: This is a closed book quiz, no notes or calculators allowed. Please turn off all cell phones, pagers, etc.

1. Use Laplace transforms to solve the initial value problem

$$y'' + 5y' + 4y = 9e^{2t}\,, \quad y(0) = 0\,, \quad y'(0) = 3\,.$$

(There's a table of Laplace transforms on the back.)

Solution: Apply the Laplace transform to the differential equation: if we write Y for $\mathcal{L}(y(t))$, it becomes

$$(s^2Y - 3) + 5sY + 4Y = \frac{9}{s-2},$$

so

$$(s^2 + 5s + 4)Y = \frac{9}{s-2} + 3 = \frac{9 + 3s - 6}{s-2} = \frac{3s+3}{s-2}.$$

So

$$Y = \frac{3s+3}{(s-2)(s^2+5s+4)} = \frac{3s+3}{(s-2)(s+1)(s+4)} = \frac{3}{(s-2)(s+4)}.$$

Use partial fractions now:

$$Y = \frac{1/2}{s-2} - \frac{1/2}{s+4}.$$

Now apply the inverse Laplace transform:

$$y = \frac{1}{2}e^{2t} - \frac{1}{2}e^{-4t}.$$