**Mathematics 135 Quiz 5** Name: Answers

February 11, 2010

**Instructions**: This is a closed book quiz, no notes or calculators allowed. Please turn off all cell phones, pagers, etc.

1. Find the solution to the initial value problem

$$y'' + 4y = 8\sin(2x)\,, \quad y(0) = 0\,, \quad y'(0) = 0\,.$$

**Solution:** This is a second order linear constant coefficient nonhomogeneous equation. The associated homogeneous equation is

$$y'' + 4y = 0,$$

which has characteristic equation $m^2 + 4 = 0$. This has roots $m = \pm 2i$, so the complementary solution $y_c$ is

$$y_c = c_1 \cos 2x + c_2 \sin 2x.$$

To find a particular solution, the first guess might be

$$y_p = A\cos 2x + B\sin 2x,$$

but for any choice of $A$ and $B$, this is a solution to the homogeneous equation, so it won't work. Multiplying by $x$ should fix this: try

$$y_p = Ax\cos 2x + Bx\sin 2x.$$

I want to plug this into the differential equation, so I calculate its derivatives:

$$\begin{aligned} y_p' &= -2Ax\sin 2x + A\cos 2x + 2Bx\cos 2x + B\sin 2x, \\ y_p'' &= -4Ax\cos 2x - 4A\sin 2x - 4Bx\sin 2x + 4B\cos 2x. \end{aligned}$$

Therefore $y_p'' + y_p$ equals $-4A\sin 2x + 4B\cos 2x$. This must equal $8\sin 2x$, which means that $A = -2$ and $B = 0$, so $y_p = -2x\cos 2x$, and the general solution is

$$y = c_1\cos 2x + c_2\sin 2x - 2x\cos 2x.$$

Now we plug in the initial conditions to find $c_1$ and $c_2$: since $y(0) = 0$, we get $c_1 = 0$, and since $y'(0) = 0$, we get $c_2 = 1$:

$$y'(x) = -2c_1\sin 2x + 2c_2\cos 2x - 2\cos 2x + 4x\sin 2x,$$

and plugging in $x = 0$ and $y' = 0$ gives $2c_2 - 2 = 0$. So the answer is

$$y = \sin 2x - 2x\cos 2x.$$

It's a good idea to check your work: plug $x = 0$ into both $y$ and $y'$ to make sure you get zero, and plug $y$ into the differential equation to make sure it's actually a solution.