Mathematics 135 Quiz 3

Name: Answers

January 21, 2010

Instructions: This is a closed book quiz, no notes or calculators allowed. Please turn off all cell phones, pagers, etc.

1. Does the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k^{3/2}}$ converge? If so, does it converge absolutely or conditionally? As always, justify your answer.

Solution: It converges absolutely: the series of absolute values is $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^{3/2}}$, which is a p-series with $p = 3/2 > 1$. Any p-series with $p > 1$ converges, so this does; therefore the original alternating series converges absolutely.

(You can use the alternating series test to show that the original series converges, but you can't tell absolute vs. conditional convergence from that.)

2. For each $n \geq 1$, compute the degree n Taylor polynomial $P_n(x)$ for the function $f(x) = x^3 + x$ at the point $x = 1$. (That is, with $a = 1$.)

Solution: We compute the derivatives of f: $f'(x) = 3x^2 + 1$, $f''(x) = 6x$, $f'''(x) = 6$, and all of the higher derivatives are zero. Now we plug into the formula for $P_n(x)$:

$$
\begin{aligned}
P_1(x) &= 1 + 4(x-1) \\
P_2(x) &= 1 + 4(x-1) + 3(x-1)^2 \\
P_3(x) &= 1 + 4(x-1) + 3(x-1)^2 + (x-1)^3 \\
P_4(x) &= 1 + 4(x-1) + 3(x-1)^2 + (x-1)^3 \\
P_n(x) &= 1 + 4(x-1) + 3(x-1)^2 + (x-1)^3 \quad \text{for all } n \geq 4
\end{aligned}
$$