Mathematics 135 Quiz 2 Name: Answers

January 14, 2010

Instructions: This is a closed book quiz, no notes or calculators allowed. Please turn off all cell phones, pagers, etc. As always, justify all of your answers.

1. Does the series $\sum_{k=1}^{\infty} \left(\frac{k+1}{k}\right)^k$ converge or diverge? If it converges, can you find the sum?

Solution: It diverges because the limit of the terms is not zero:

$$\lim_{k\to\infty} \left(\frac{k+1}{k}\right)^k = \lim_{k\to\infty} \left(1+\frac{1}{k}\right)^k = e.$$

In any series, if the limit of the terms is not zero, the series must diverge.

2. Does the series $\sum_{k=0}^{\infty} \frac{1}{2^{k+3}}$ converge or diverge? If it converges, can you find the sum?

Solution: It converges, and we can evaluate the sum using basic properties of series and appealing to the formula for a geometric series:

$$\begin{aligned}\sum_{k=0}^{\infty} \frac{1}{2^{k+3}} &= \frac{1}{2^3}\sum_{k=0}^{\infty} \frac{1}{2^k} = \frac{1}{8}\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k \\ &= \frac{1}{8}\,\frac{1}{1-1/2} = \frac{1}{8}\cdot 2 = \frac{1}{4}.\end{aligned}$$