Mathematics 135 Quiz 1

Name: _____Answers_____

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Instructions: This is a closed book quiz, no notes or calculators allowed. Please turn off all cell phones, pagers, etc.

1. Evaluate the limit $\displaystyle\lim_{n\to\infty} \ln\left(\frac{2n}{n+1}\right)$. Justify all of your steps, carefully stating which theorems and properties of limits you use.

Solution: First,

$$\lim_{n\to\infty} \ln\left(\frac{2n}{n+1}\right) = \lim_{n\to\infty} \ln\left(\frac{2}{1+\frac{1}{n}}\right) \quad \text{(by simple algebra)}.$$

Next, as $n \to \infty$, $1/n \to 0$ (done in class), and clearly $1 \to 1$, so, since the limit of a sum is the sum of the limits, $1 + \frac{1}{n} \to 1$. Since this limit is nonzero, then the limit of the quotients is the quotient of the limits:

$$\frac{2}{1+\frac{1}{n}} \longrightarrow \frac{2}{1} = 2.$$

Finally, since the function ln is defined and continuous for all positive numbers,

$$\lim_{n\to\infty} \ln\left(\frac{2}{1+\frac{1}{n}}\right) = \ln\left(\lim_{n\to\infty} \frac{2}{1+\frac{1}{n}}\right) = \ln(2).$$

So the answer is $\ln 2$.