Midterm 2 review

Topics:

- linear first order equations and their integrating factors
- Picard iteration
- linear degree n equations, linear independence, form of the general solution in both the homogeneous and non-homogeneous cases
- solving linear constant coefficient homogeneous equations using the characteristic equation
- finding a second solution to a linear homogeneous equation using reduction of order
- solving linear constant coefficient nonhomogeneous equations using the method of undetermined coefficients and using variation of parameters
- Laplace transforms and inverse Laplace transforms: basic formulas, using tables, discontinuous forcing functions, the Dirac δ-function
- solving linear constant coefficient equations using Laplace transforms
- power series solutions, method of Frobenius

An old Math 135 midterm

(1) **(10 points)** Find the general solution of the differential equation $y'' + y = 2t^2 + t$.

(2) **(5 points)** Find a particular solution of the differential equation $y'' - y = (1+t^2)^{-1}$. Express your answer in terms of integrals, and do not attempt to evaluate these integrals.

(3) **(5 points)** Is there a differential equation of the form $y'' + p(t)y' + q(t)y = 0$, with p and q continuous on the open interval $-1 < t < 1$, for which the functions $y = t^4$ and $y = t^6$ are both solutions? Explain your answer.

(4) **(10 points)** The solution of the initial value problem

$$y'' + (t+1)y' + y = 0\,, \quad y(0) = 1 \quad y'(0) = 0$$

can be expressed in the form $y = \sum_{k=0}^{\infty} a_k t^k$. Find the recursion formula for the coefficients.

(5) (a) **(2 points)** Find $\mathcal{L}^{-1}\left[\frac{1}{(s-1)^2}\right]$.

(b) **(3 points)** Use the result of part (a) to find $\mathcal{L}^{-1}\left[\frac{e^{-3s}}{(s-1)^2}\right]$.

(c) **(5 points)** Use the results of parts (a) and (b) to solve the initial value problem

$$y'' - 2y' + y = \delta(t-3)\,, \quad y(0) = 0\,, \quad y'(0) = 1\,.$$

(6) **(10 points)** By computing Laplace transforms, show that the solution of the initial value problem

$$y'' - y = f(t)\,, \quad y(0) = y'(0) = 0$$

can be written as a convolution of f with another function. What is that other function?