

## Some Laplace transforms

Definition of Laplace transform:  $\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t) dt$

$f(t)$	$F(s)$
0	0
1	$\frac{1}{s}, \quad s > 0$
$t^n, \quad n = 0, 1, 2, \dots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
$e^{at}$	$\frac{1}{s-a}, \quad s > a$
$\sin(at)$	$\frac{a}{s^2 + a^2}, \quad s > 0$
$\cos(at)$	$\frac{s}{s^2 + a^2}, \quad s > 0$
$e^{ct} f(t)$	$F(s-c)$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$e^{at} \sin(bt)$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$
$e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
$-tf(t)$	$F'(s)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$t \sin at$	$\frac{2as}{(s^2 + a^2)^2}$
$t \cos at$	$\frac{s^2 - a^2}{(s^2 + a^2)^2}$

$f(t)$	$F(s)$
$\sinh(at)$	$\frac{a}{s^2 - a^2}, \quad s >  a $
$\cosh(at)$	$\frac{s}{s^2 - a^2}, \quad s >  a $
$t^p$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$

$$\text{(where } \Gamma(k) = \int_0^\infty e^{-x} x^{k-1} dx \text{)}$$

$x^{-1/2}$	$\frac{\sqrt{\pi}}{\sqrt{s}}$
$\int_0^t f(u) du$	$\frac{F(s)}{s}$
$\frac{f(t)}{t}$	$\int_s^\infty F(p) dp$
$\int_0^t f(t-u)g(u)du$	$F(s)G(s)$
$u(t-c)$	$e^{-cs} \frac{1}{s}$
$u(t-c)f(t-c)$	$e^{-cs} F(s)$
$\delta(t-c)$	$e^{-cs}$