Math 135: Homework 9

Due Thursday, March 4

(1) Show that for any vectors $\mathbf{a}$ and $\mathbf{b}$,

$$\big|\,\|\mathbf{a}\| - \|\mathbf{b}\|\,\big| \le \|\mathbf{a} - \mathbf{b}\|$$

(Hint: $\mathbf{a} = (\mathbf{a} - \mathbf{b}) + \mathbf{b}$.)

(2) Let T denote the tetrahedron centered at the origin O with vertices at the points $P_1(1, 1, 1)$, $P_2(-1, -1, 1)$, $P_3(1, -1, -1)$, and $P_4(-1, 1, -1)$. Using vector methods, find the cosine of the angles $\angle P_i O P_j$ for all $i \ne j$. What are the approximate angles (in degrees)?

(3)

    (a) Suppose that $\mathbf{a} \cdot \mathbf{b} = 0$ and $\mathbf{a} \times \mathbf{b} = \mathbf{0}$. Show that either $\mathbf{a} = \mathbf{0}$ or $\mathbf{b} = \mathbf{0}$.

    (b) Now suppose that $\mathbf{a} \ne \mathbf{0}$. If $\mathbf{c}$ and $\mathbf{d}$ are vectors for which

$$\mathbf{a} \cdot \mathbf{c} = \mathbf{a} \cdot \mathbf{d} \text{ and } \mathbf{a} \times \mathbf{c} = \mathbf{a} \times \mathbf{d},$$

    use the result of (a) to prove that $\mathbf{c} = \mathbf{d}$.