Math 135: Homework 7

Due Thursday, February 18

(1) Recall that the Laplace transform of the function $u(t-c)f(t-c)$, $c \geq 0$ is $e^{-cs}F(s)$, where F is the Laplace transform of f. Use this fact to compute the inverse Laplace transform of $\dfrac{1-e^{-2s}}{s^2+1}$.

(2) Let m and k be positive constants, and consider the differential equation

$$my'' + ky = f(t)\,.$$

Suppose that $y(0) = y'(0) = 0$, and let $f(t)$ be given by

$$f(t) = \begin{cases} 0 & \text{for } t < t_0 \\ F_0 & \text{for } t_0 \leq t \leq t_0 + \Delta t \\ 0 & \text{for } t > t_0 + \Delta t, \end{cases},$$

for some constants $t_0, \Delta t > 0$. Use Laplace transform techniques to find $y(t)$.

(3) Let $T > 0$ be a fixed constant, and let f be a piecewise continuous function satisfying the condition $f(t+T) = f(t)$ for all $t \geq 0$. Such a function is said to be *periodic* with *period* T. Show that in this case,

$$\mathcal{L}[f](s) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt.$$

(4) Use (3) to compute the Laplace transform of the *sawtooth wave*

$$f(t) = \begin{cases} t & \text{for } 0 \leq t < 1, \\ f(t-1) & \text{for } t \geq 1. \end{cases}$$