Math 135: Homework 6

Due Thursday, February 11

1. In the following, you are given a differential equation and one solution of it. Use *reduction of order* to use find the general solution. We discussed this in class on Monday: try a solution of the form $y = u(x)y_1(x)$ and solve for the unknown function $u(x)$. (This is also discussed in the book: in the constant coefficient case, see Lesson 20.C, and with nonconstant coefficients but using slightly different notation, in Lesson 23.)

   (a) $x^2y'' - x(x+2)y' + (x+2)y = 0$, $y_1(x) = x$.

   (b) $xy'' - (x+2)y' + 2y = 0$, $y_1(x) = e^x$.

   (c) $x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$, $y_1(x) = x^{-1/2}\sin x$.

2. Fix an interval $I = (a, b)$. Let $y_1(x)$ and $y_2(x)$ be solutions of $y'' + p(x)y' + q(x)y = 0$ on I, where $p(x)$, $q(x)$ are continuous functions on I. Show that if there is a point in I where both y_1 and y_2 both vanish or where both have maxima or minima, then one of y_1 and y_2 is a multiple of the other.