Math 135: Homework 5

Due Thursday, February 4

1. Fix a real constant α. Show that $\displaystyle\sum_{k=0}^{\infty} \frac{\sin \alpha k}{2^k}$ converges. Evaluate the sum exactly (in terms of α) by using the fact that $\sin \alpha k = \mathrm{Im}(e^{i\alpha k})$. **Hints:** You may assume that all of the series we've been discussing work equally well with complex numbers as with real numbers. Also, note that for any real numbers a and b, we have $\frac{1}{a+ib} = \frac{a}{a^2+b^2} - i\frac{b}{a^2+b^2}$.

2. Fix a real constant λ and consider the second order differential equation

$$y'' + \lambda y = 0.$$

Let a and b be real numbers.

   (a) Check that $y = ax + b$ satisfies the equation when $\lambda = 0$.

   (b) Check that $y = a\cos(\sqrt{\lambda}\, x) + b\sin(\sqrt{\lambda}\, x)$ satisfies the equation when $\lambda > 0$.

   (c) Check that $y = a\cosh(\sqrt{-\lambda}\, x) + b\sinh(\sqrt{-\lambda}\, x)$ satisfies the equation when $\lambda < 0$.