(1) (15 points) The two curves $\mathbf{r}(t) = t\hat{\mathbf{i}} + t^2\hat{\mathbf{j}} + t^3\hat{\mathbf{k}}$ and $\mathbf{R}(s) = s^2\hat{\mathbf{i}} + s^3\hat{\mathbf{j}} + s^4\hat{\mathbf{k}}$ intersect at the point $(1, 1, 1)$.

(a) Find parametric equations for each of the tangent lines to these curves at $(1, 1, 1)$.

(b) Find the angle between the two tangent lines to the curves at that point.

(c) Find the equation of the plane containing these two tangent lines.

(2) (15 points) Consider the initial value problem $y'' + y = \sin(\omega t)$, $y(0) = y'(0) = 0$.

(a) Find the solution for $\omega \neq 1$.

(b) Find the solution for $\omega = 1$.

(3) (10 points) Solve the initial value problem $y'' + y = \frac{1}{\cos(t)}$, $y(0) = y'(0) = 0$, $|t| < \pi/2$.

(4) (10 points) Consider the power series $\displaystyle\sum_{k=0}^{\infty} \frac{x^{2k+1}}{3^k \ln(2k+2)}$.

(a) Find its radius of convergence.

(b) Find its interval of convergence.

(5) (10 points) Solve the initial value problem $y'' - ty = 0$, $y(0) = y'(0) = 0$ using power series.

(6) (5 points) Express the solution of the initial value problem $y'' - y = \tan(t)$, $y(0) = y'(0) = 0$, as the convolution of two functions. (*Do not attempt to evaluate the convolution.*)

(7) (5 points) Evaluate $\displaystyle\lim_{x \to 0} \frac{x\sin(x) - \sin(x^2)}{\sin(2x^4)}$ in any way you wish.

(8) (10 points) Let $y = y(t)$ be a function that satisfies the identity $y(t) = \displaystyle\int_0^t \frac{s}{\cos(y(s))} ds$.

(a) Find an initial value problem that the function $y = y(t)$ satisfies.

(b) Solve the initial value problem to find $y(t)$.