

Extra problem for homework 8

Suppose that a and b positive integers which are coprime: their gcd is 1. At a certain store, you can buy a certain product, say pomelos, in boxes of size a or b .

Theorem. *There is an integer N so that for any integer $k \geq N$, you can buy exactly k pomelos. That is, if a and b are positive integers which are coprime, then there exists an integer N so that for all integers $k \geq N$, there exist non-negative integers m and n with*

$$k = ma + nb.$$

Your goal is to prove this theorem and also to come up with a formula, or some sort of expression, for N in terms of a and b .

If you can prove that your formula is correct for $a = 2$ and $a = 3$, you will get full credit. If you can prove the theorem and/or prove that your formula is correct in general, you will get some bonus points.