Mathematics 134 Quiz 7   Name: _____Answers_____

November 19, 2009

Instructions: This is a closed book quiz, no notes or calculators allowed. Please turn off all cell phones, pagers, etc.

1. Compute $\displaystyle\int_e^3 \frac{dx}{x\,(\ln x)^2}$.

> **Solution:** Let $u = \ln x$; then $du = dx/x$, so the integral becomes
> $$\int \frac{du}{u^2} = \frac{-1}{u} = \frac{-1}{\ln x}\bigg|_e^3 = -\frac{1}{\ln 3} + \frac{1}{\ln e} = 1 - \frac{1}{\ln 3}.$$

2. What is the maximum value of the function $\sqrt[x]{x}$? (Assume that x is positive.)

> **Solution:** We have
> $$\sqrt[x]{x} = x^{1/x} = e^{\ln(x^{1/x})} = e^{\frac{1}{x}\ln x}.$$
> Take the derivative: you get
> $$\left(\frac{1}{x}\ln x\right)' e^{\frac{1}{x}\ln x} = \left(-\frac{1}{x^2}\ln x + \frac{1}{x^2}\right) e^{\frac{1}{x}\ln x} = \frac{1}{x^2}(1 - \ln x)e^{\frac{1}{x}\ln x}.$$
> The first and third factors here are always positive, so the sign of the derivative is controlled by $1 - \ln x$. This quantity is positive when $\ln x < 1$, which is to say, when $x < e$, it's zero when $x = e$, and it's negative when $x > e$. Therefore the function has a single critical point, $x = e$, and this point gives a maximum by the first derivative test. The maximum value is therefore $\sqrt[e]{e}$.