Mathematics 134 Quiz 5   Name: _____Answers_____

November 5, 2009

Instructions: This is a closed book quiz, no notes or calculators allowed. Please turn off all cell phones, pagers, etc.

1. Use upper and lower sums to show that

$$\frac{2}{3} < \int_1^3 \frac{dx}{x} < 2.$$

> **Solution:** Partition the interval $[1, 3]$ into a single piece. Since the function $y = 1/x$ is decreasing on this interval, the maximum value occurs when $x = 1$ and the minimum when $x = 3$: the maximum is 1 and the minimum is $1/3$. The width of the interval is 2, so the lower sum is $2/3$ and the upper sum is 2. The integral lies between these numbers:
> $$\frac{2}{3} < \int_1^3 \frac{dx}{x} < 2.$$

2. Let $F(x) = \int_0^x (t-1)(t^2 - 4)dt$. Find the critical points of $F(x)$ and determine the intervals on which it is increasing and on which it is decreasing.

> **Solution:** By the fundamental theorem of calculus, the derivative of $F(x)$ is
>
> $$F'(x) = (x-1)(x^2 - 4) = (x-1)(x-2)(x+2).$$
>
> The critical points are therefore $x = 1$, $x = 2$ and $x = -2$. $F(x)$ is increasing on $(2, \infty)$ and $(-2, 1)$. It is decreasing on $(1, 2)$ and $(-\infty, -2)$.