Mathematics 134 Quiz 2

Name: Answers

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Instructions: This is a closed book quiz, no notes or calculators allowed. Please turn off all cell phones, pagers, etc.

1. Let f be continuous at c. Prove that if $f(c) > 0$, then there is a $\delta > 0$ so that $f(x) > 0$ for all $x \in (c - \delta, c + \delta)$.

Solution: By the definitions of limit and continuity, for any $\varepsilon > 0$, there is a $\delta > 0$ so that if $|x - c| < \delta$, then $|f(x) - f(c)| < \varepsilon$. Choose $\varepsilon = f(c)/2$: this is positive since $f(c)$ is. Find a corresponding δ; then x is in $(c - \delta, c + \delta)$ if and only if $|x - c| < \delta$, and if this holds, then $|f(x) - f(c)| < f(c)/2$, which means that $f(c)/2 < f(x) < 3f(c)/2$. In particular, $f(x)$ is positive for all of these values of x.

2. Let a and b be nonzero constants, and compute $\lim_{x \to 0} \dfrac{\sin ax}{bx}$, justifying your answer using the various theorems on limits.

Solution: We compute:

$$\lim_{x \to 0} \frac{\sin ax}{bx} = \lim_{x \to 0} \frac{a}{b} \frac{\sin ax}{ax}$$

We pull the constant out of the limit:

$$= \frac{a}{b} \lim_{x \to 0} \frac{\sin ax}{ax}$$

We make a change of variables $y = ax$:

$$= \frac{a}{b} \lim_{y \to 0} \frac{\sin y}{y}$$

and we evaluate the limit:

$$= \frac{a}{b} \cdot 1 = \frac{a}{b}.$$