Math 134: Homework 5

Due October 29

1. Suppose that $f(x)$ is concave up on an interval I. Show that for any $a, b \in I$ with $a < b$,

$$f(x) \leq \frac{f(b) - f(a)}{b - a}(x - a) + f(a)$$

   for all $x \in [a, b]$. (That is, $f(x)$ lies below the chord from $(a, f(a))$ to $(b, f(b))$.)

2. Use the result (not your proof, just the result) from part 1 to show that if $f(x)$ is concave down, then for any $a, b \in I$ with $a < b$,

$$f(x) \geq \frac{f(b) - f(a)}{b - a}(x - a) + f(a)$$

   for all $x \in [a, b]$.

   (Hint: if $f(x)$ is concave down, can you find a related function which is concave up?)

3. Bonus: Prove the converse: suppose that for all $a, b \in I$ with $a < b$,

$$f(x) \leq \frac{f(b) - f(a)}{b - a}(x - a) + f(a)$$

   for all $x \in [a, b]$. Prove that $f(x)$ is concave up on I.

   Do you need to assume that $f(x)$ is differentiable in I, or can you deduce it? Same for continuity?