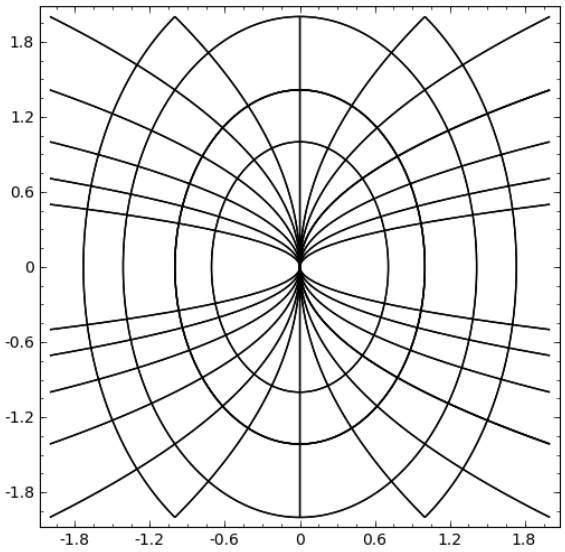
Math 134: Homework 4

Due October 22

1. Two curves are called *orthogonal* if, wherever they intersect, they intersect at a right angle. (That is, their tangent lines are perpendicular at each point of intersection.) Two families of curves are *orthogonal trajectories* if each curve from one family is orthogonal to each curve from the other family.

   Show that the family of parabolas $x = ay^2$ and the family of ellipses $x^2 + \frac{1}{2}y^2 = b$ (for all real numbers a and all positive real numbers b) are orthogonal trajectories.



2. Let f be a function that is differentiable for all $x \geq 0$. Suppose that $f'(x)$ satisfies the condition

$$a \leq f'(x) \leq b \text{ for all } x > 0.$$

   Show that for all $x > 0$,

$$f(0) + ax \leq f(x) \leq f(0) + bx\,.$$

   (Hint: use the Mean Value Theorem.)